Bottom-up thermalization from holography?

Stefan Stricker

TU Vienna

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D. Steineder, SS, A. Vuorinen, Phys. Rev. Lett. 110, 101601 (2013)
R. Baier, SS, O. Taanila, A. Vuorinen,, Phys. Rev. D 86, 081901(R) (2012)
R. Baier, SS, O. Taanila, A. Vuorinen, JHEP 1207 (2012) 094

Motivation

quark gluon plasma

- produced in heavy collisions at RHIC and LHC
- behaves as a strongly coupled liquid
- thermalization process not well understood

goals

- gain insight into the thermalization process
- modification of production rates of photons/dileptons
- which modes thermalize first: top-down or bottom-up ?
- dependence on coupling strength

strategy

- SYM where strong and weak coupling regimes are accessible

Thermalization scenarios

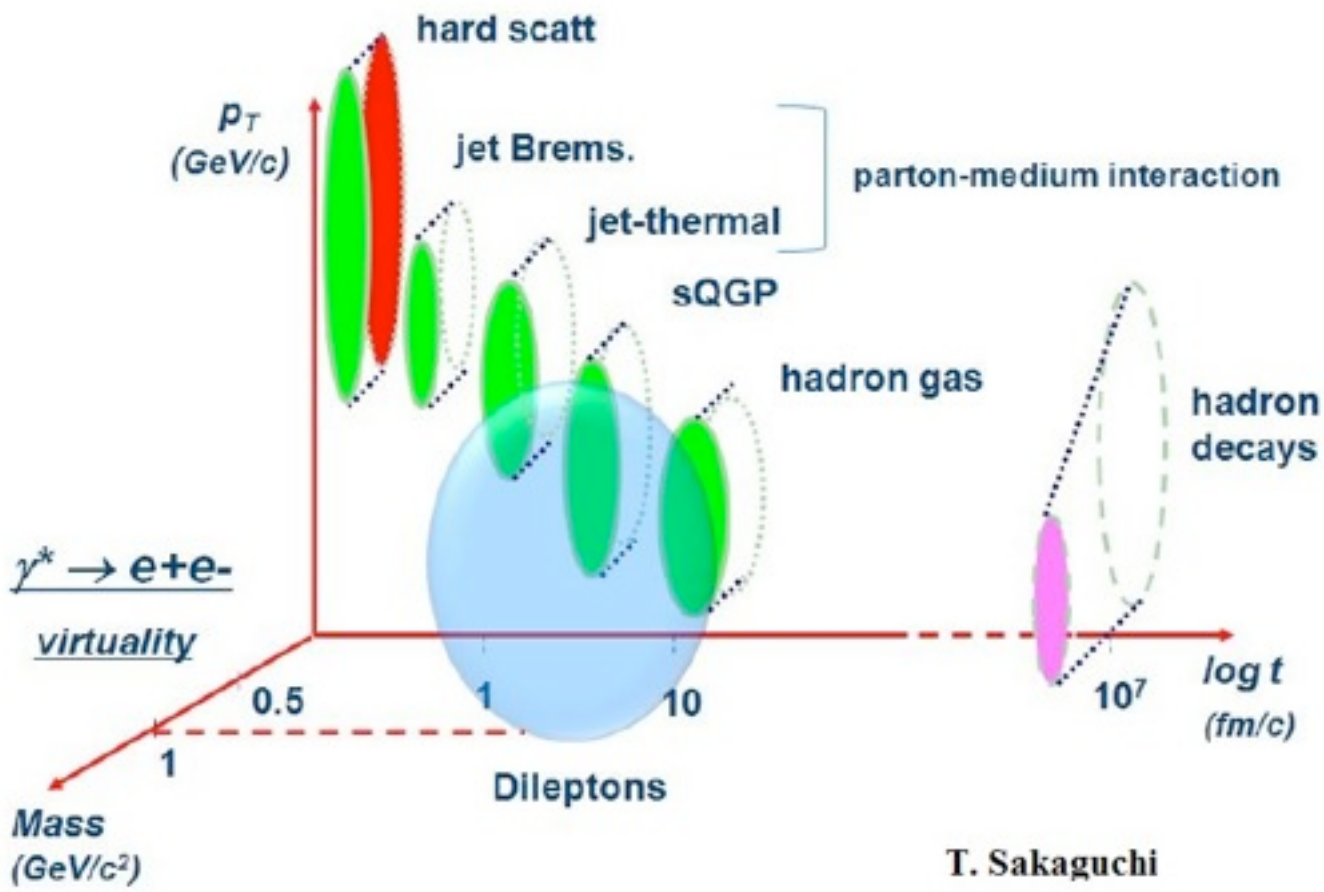
bottom up scenario

- at weak coupling
- scattering processes
  - in the early stages many soft gluons are emitted which then thermalize the system (*Baier et al (2001)*)
- driven by instabilities
  - instabilities isotropize the momentum distributions more rapidly than scattering processes (*Kurkela, Moore (2011)*)

top down scenario

- at strong coupling
- UV modes thermalize first
- in AdS calculations, follows naturally from causality

Photon emission in heavy ion collisions



photons are emitted at all stages of the collision

- initial hard scattering processes: quark anti-quark annihilation:
  - on-shell photon or virtual photon → dilepton pair
- strongly coupled out of equilibrium phase: no quasiparticle picture
- additional (uninteresting) emissions from charged hadron decays

Probing the plasma

probing the plasma

- once produced photons/dileptons stream through the plasma almost unaltered
- provide observational window in the thermalization process of the plasma

quantity of interest

- spectral density : $\chi^{\mu}_{\mu} = -2\mathrm{Im}(\Pi^{\mathrm{ret}})^{\mu}_{\mu}(k_0)$
- number of photons emitted with given momentum

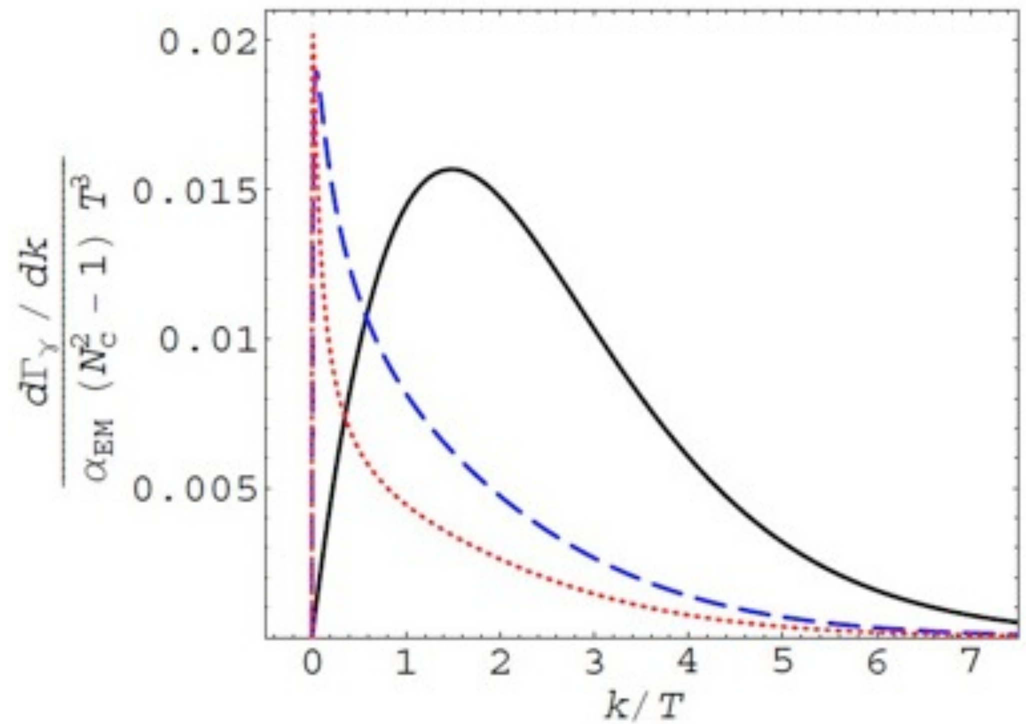
fluctuation dissipation theorem

$$\eta^{\mu\nu}\Pi^{<}_{\mu\nu}(\omega) = -2n_B(\omega)\mathrm{Im}(\Pi^{ret})^{\mu}_{\mu}(\omega) = n_B(\omega)\chi(\omega)$$

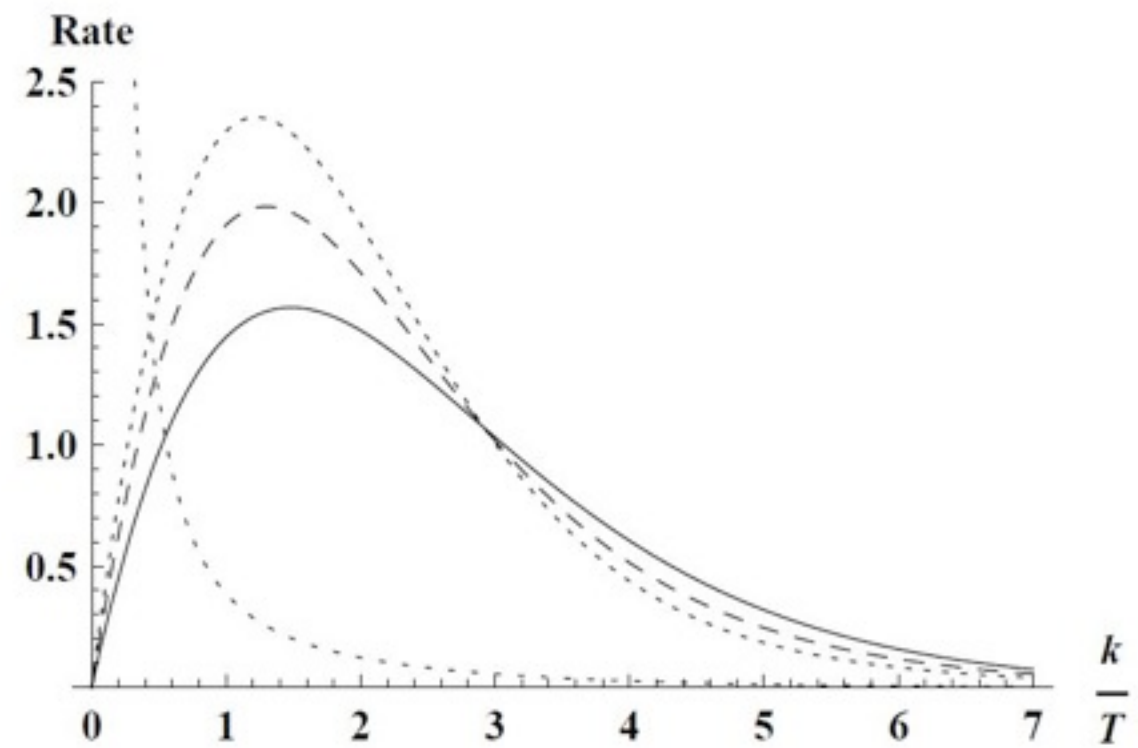
production rate

$$k^0\frac{d\Gamma_\gamma}{d^3k} = \frac{\alpha}{4\pi^2}\eta^{\mu\nu}\Pi^{<}_{\mu\nu}(\omega = k^0)$$

Photon emission in equilibrium SYM plasma



Huot et al (2006)



Hassanain, Schvellinger (2012)

perturbative result

- increasing the coupling: slope at k=0 decreases, hydro peak broadens and moves right

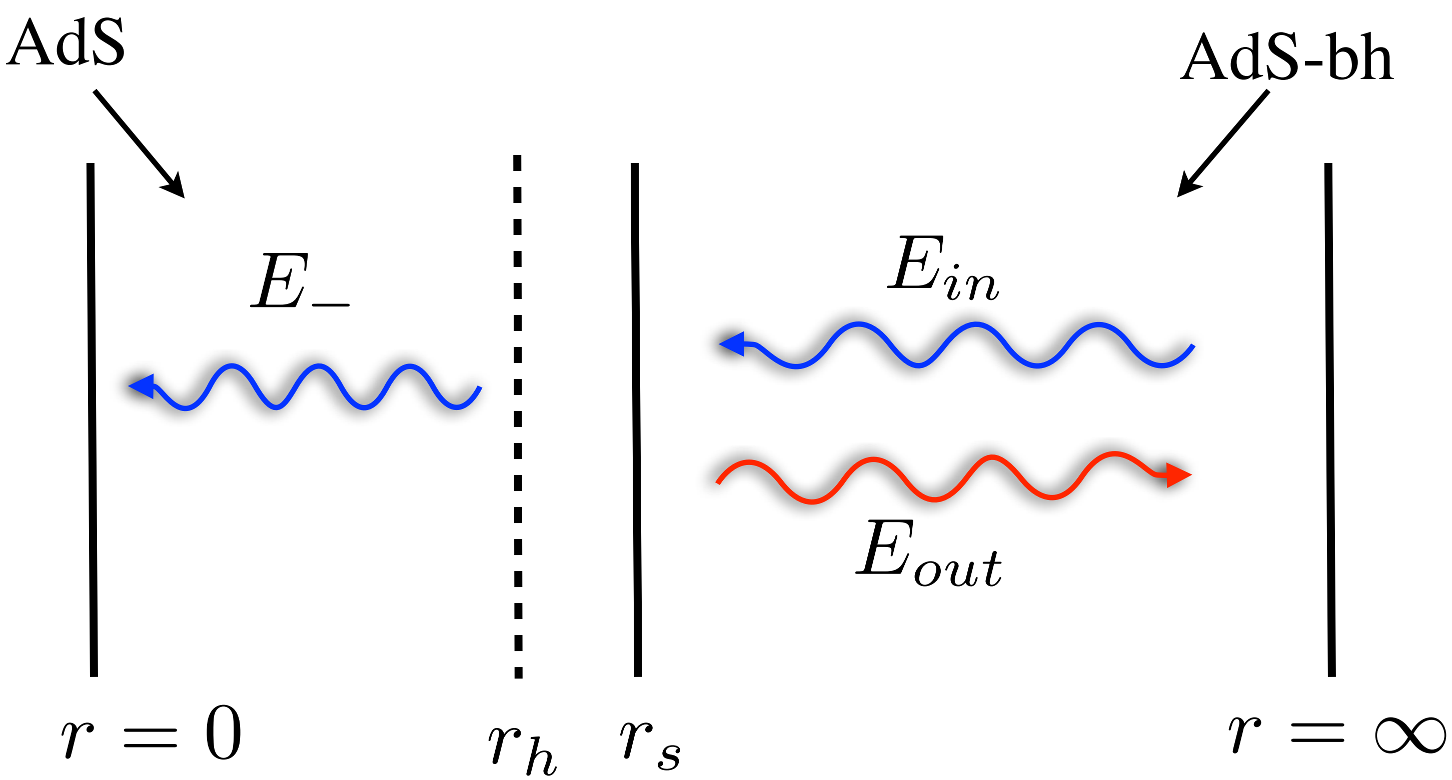
strong coupling result

- decreasing coupling from $\lambda = \infty$: peak sharpens and moves left

Out of equilibrium

- equilibrium picture in SYM fairly complete
- how does photon/dilepton production get modified out of equilibrium
- can one access thermalization at finite coupling ?

The falling shell setup



Danielsson, Keski-Vakkuri, Kruczenski (1999)

outside and inside spacetime

- metric: $$ds^2 = \frac{(\pi T L)^2}{u}\left(f(u)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{L^2}{4u^2 f(u)}du^2 \qquad u = \frac{r_h^2}{r^2}$$

$$f(u) = \begin{cases} f_+(u) = 1 - u^2\,, & \text{for } u > 1 \\ f_-(u) = 1\,, & \text{for } u < 1 \end{cases}\,,$$

outside solution

$$E_+ = c_+ E_{in} + c_- E_{out}$$

matching condition

Israel junction condition

- extrinsic curvatures match across the shell

$$[K_{ij}] - [K]g_{ij} = 0, \qquad [K_{ij}] = K^+_{ij} - K^-_{ij}$$

- can be also adapted for other fields

Fourier transformation

- discontinuity in the time coordinate

$$\frac{dt_-}{dt_+} = \sqrt{\frac{f_+}{f_-}} \equiv \sqrt{f_m} \quad \Rightarrow \quad \int dt_+ e^{i\omega_+ t_+} = \frac{1}{\sqrt{f_m}} \int dt_- e^{\frac{i\omega_+ t_-}{\sqrt{f_m}}},$$

- identification: $\omega_- = \omega_+/\sqrt{f_m}$

matching condition:

$$\begin{aligned} E_-(\omega_-)|_{u_s} &= \sqrt{f_m} E_+(\omega_+)|_{u_s}, \\ E'_-(\omega_-)|_{u_s} &= f_m E'_+(\omega_+)|_{u_s}. \end{aligned} \quad \Longrightarrow \quad \frac{c_-}{c_+}\Big|_{u_s}$$

quasistatic approximation:

- energy scale of interest >> characteristic time scale of shell's motion

equation of motion

equation of motion for transverse electric field

$$E'' + \frac{f'}{f}E' + \frac{\hat{\omega}^2 - \hat{q}^2 f}{u f^2}E = 0, \qquad E_z \equiv F_{tz}$$

$$\hat{\omega} \equiv w/(2\pi T), \quad \hat{q} \equiv q/(2\pi T) \qquad T = \frac{r_h}{\pi}$$

- this equation is solved numerically by the ansatz:

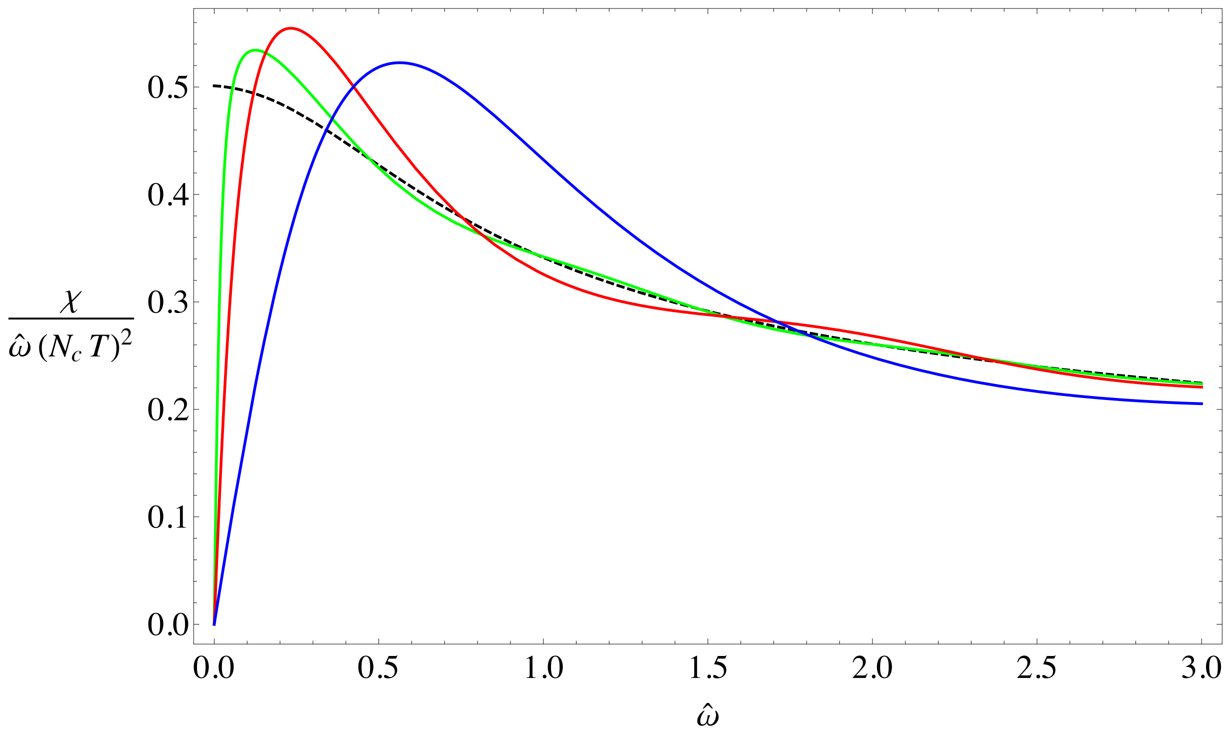
$$E_{\substack{\text{in}\\\text{out}}}(u, \hat{\omega}, \hat{q}) = (1-u)^{\mp\frac{i\hat{\omega}}{2}} y_{\substack{\text{in}\\\text{out}}}(u)$$

retarded correlator

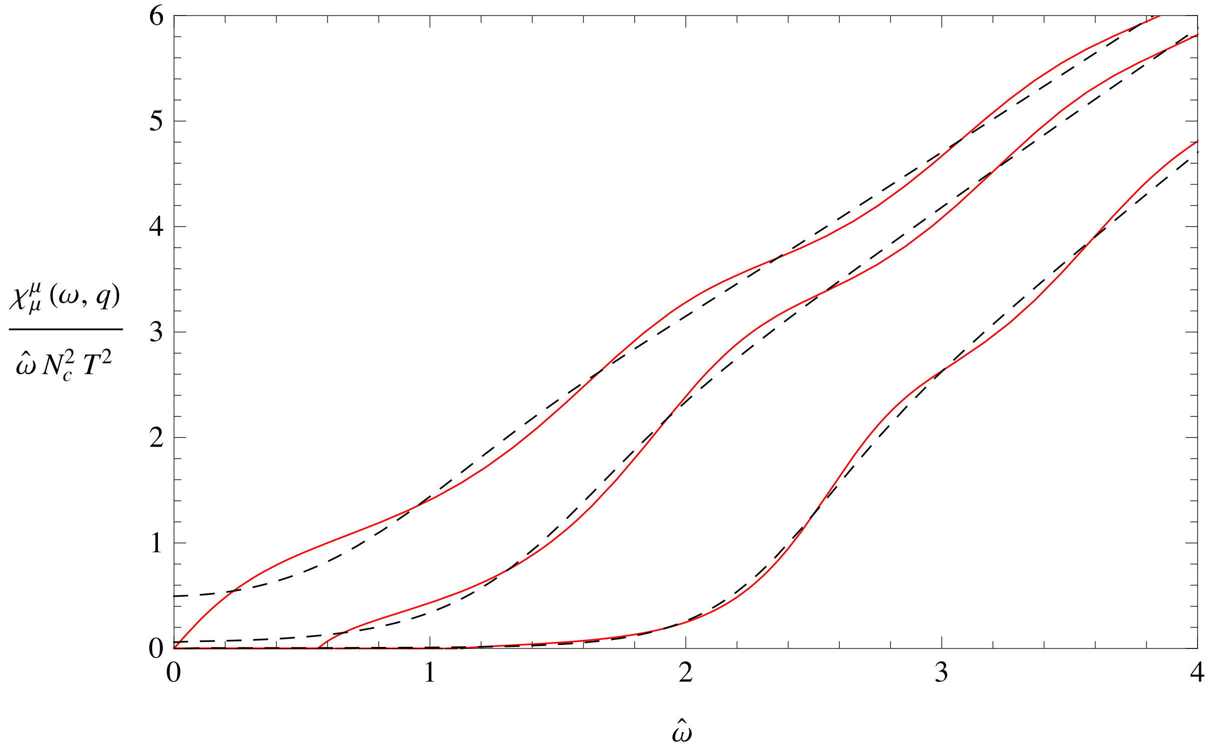
$$\Pi(\omega, \mathbf{q}) = -\frac{N_c^2 T^2}{8} \lim_{u\to 0} \frac{E'(u,Q)}{E(u,Q)} = -\frac{N_c^2 T^2}{8} \Pi_{therm} \frac{1 + \frac{c_-}{c_+}\frac{E'_{out}}{E'_{in}}}{1 + \frac{c_-}{c_+}\frac{E_{out}}{E_{in}}}$$

- reproduce thermal case: $\lim_{r_s \to r_h} \frac{c_-}{c_+} \to 0$
- behaviour of c_-/c_+ crucial for out of equilibrium dynamics

Photon & dilepton spectral density



photon spectral density for r_s/r_h =1.1, 1.01,1.001



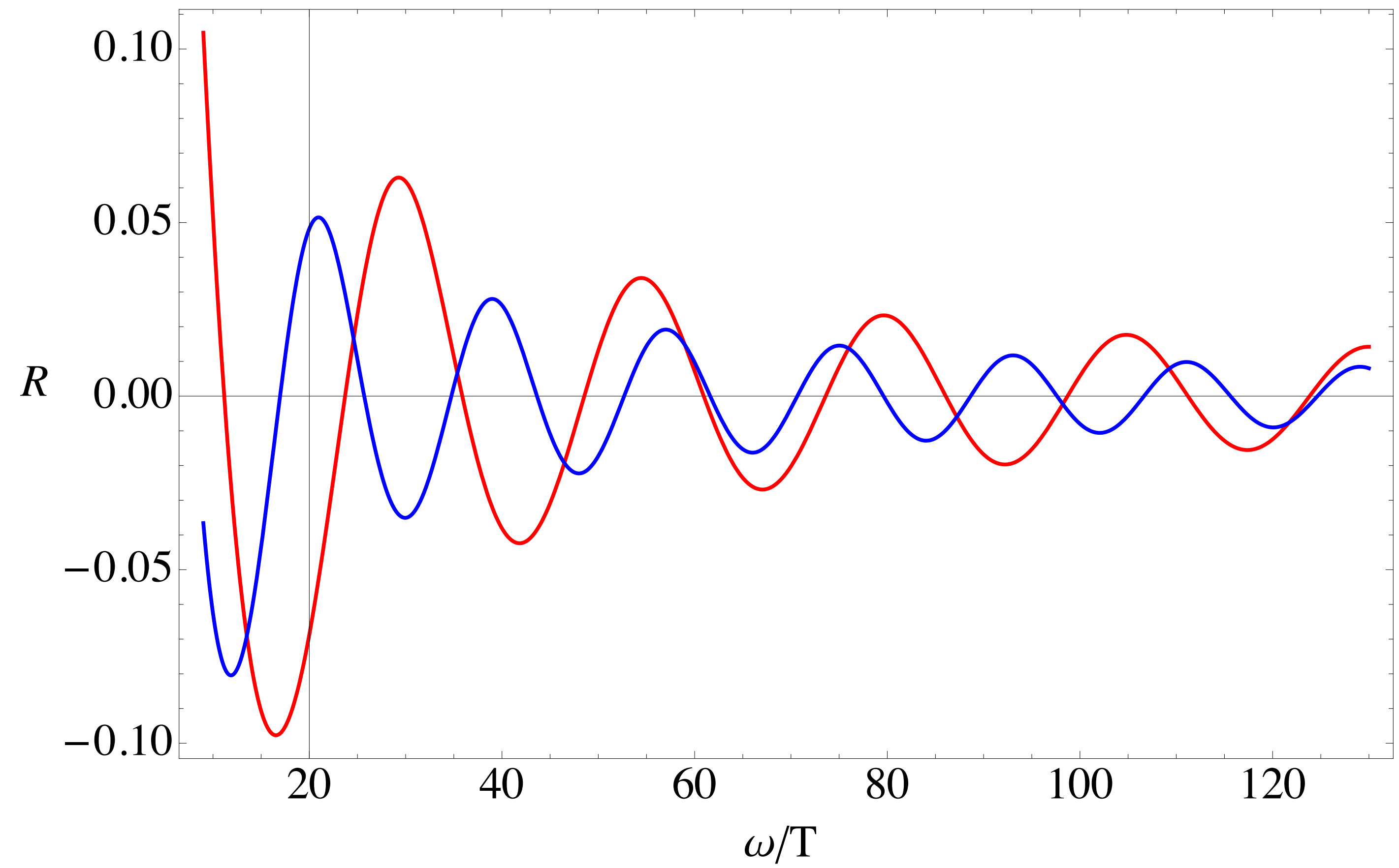
dilepton spectral density for r_s/r_h =1.01 and q=0,1,2

- out of equilibrium effect: oscillations around thermal value
- as the shell approaches the horizon equilibrium is reached

Thermalization at infinite coupling: photons

- relative deviation from thermal equilibrium

$$R(\hat{\omega}) = \frac{\chi(\hat{\omega}) - \chi_{th}(\hat{\omega})}{\chi_{th}(\hat{\omega})}$$



relative deviation R for r_s=1.01, 1.1

- thermalization: increase in frequency and decrease in amplitude

- top down thermalization: highly energetic modes are closer to equ. value

$$\chi(\hat{\omega}) \approx \hat{\omega}^{\frac{2}{3}} \left(1 + \frac{f_1(u_s)}{\hat{\omega}}\right), \qquad R \approx \frac{1}{\hat{\omega}}$$

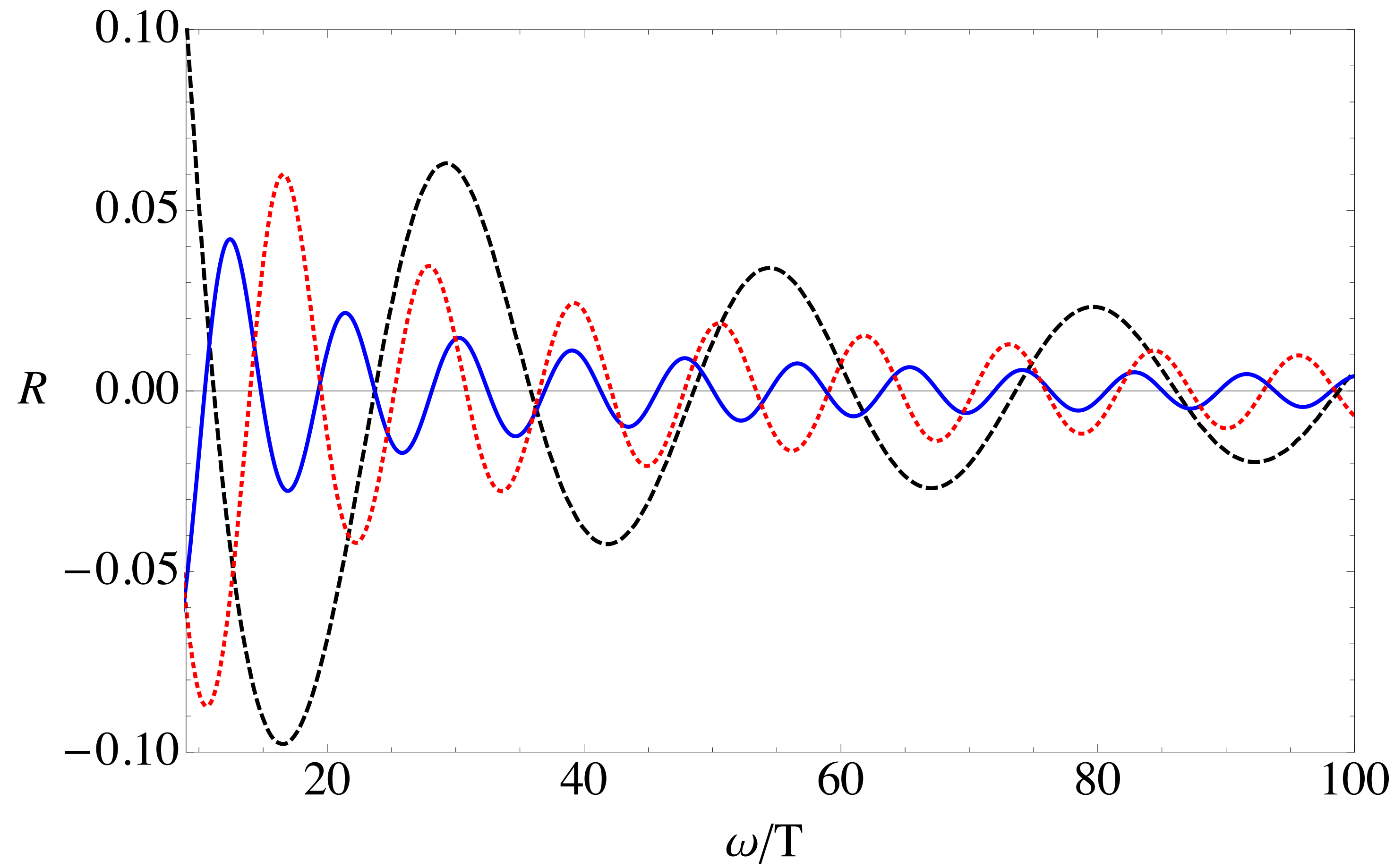
Thermalization depending on the virtuality

- virtuality

$$v = \frac{\hat{\omega}^2 - \hat{q}^2}{\hat{\omega}^2}$$

- parametrize

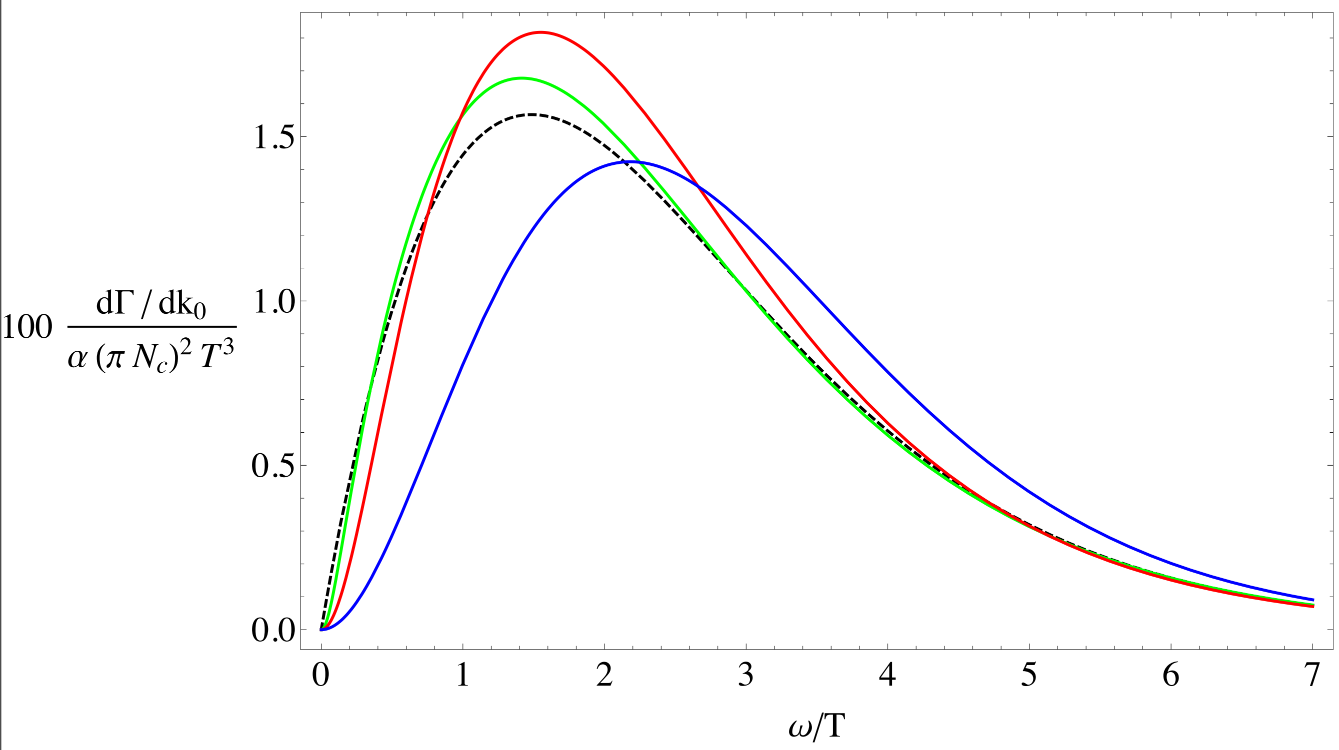
$$q = c\,\hat{\omega}$$



relative deviation R for r_s/r_h=1.01 and c=1, 0.7, 0

- thermalization depends on the virtuality
- photons are last to thermalize
- same conclusion was reached in other models of thermalization
  (Arnold et al; Chesler and Teaney)

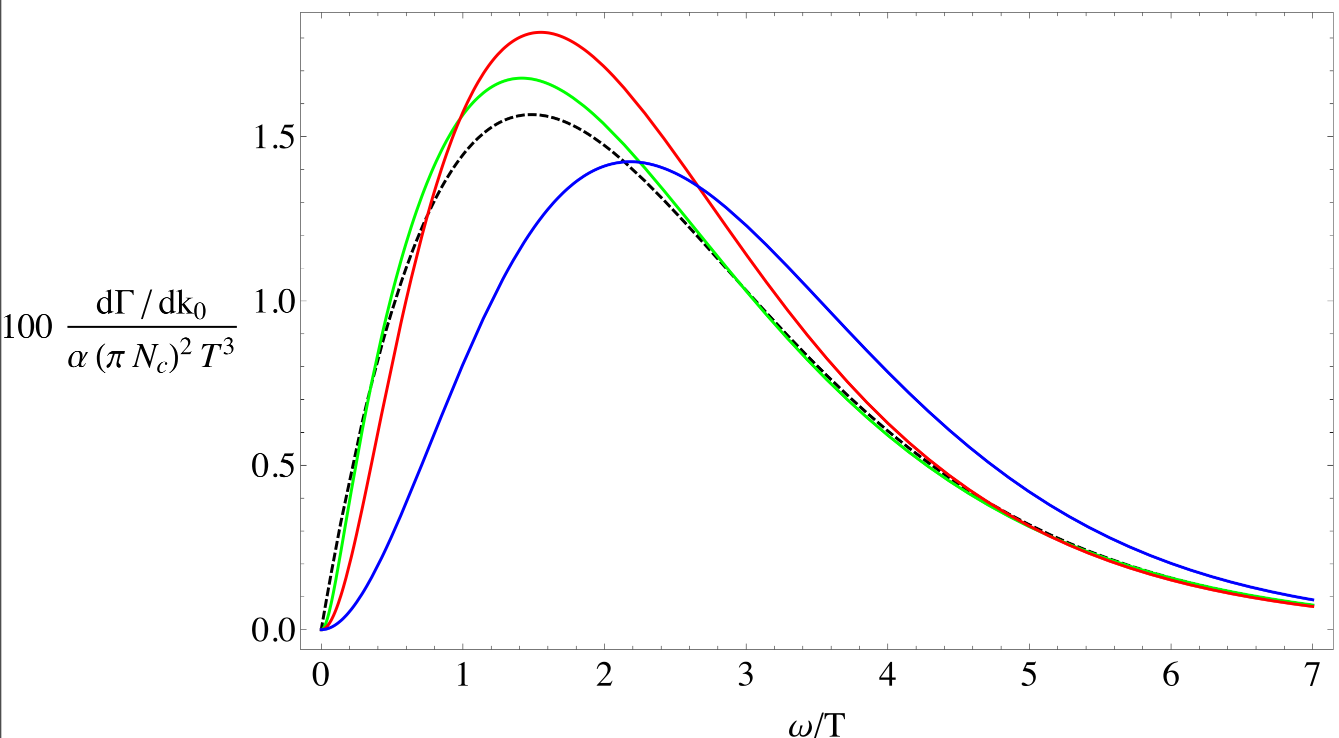
Photon production rate



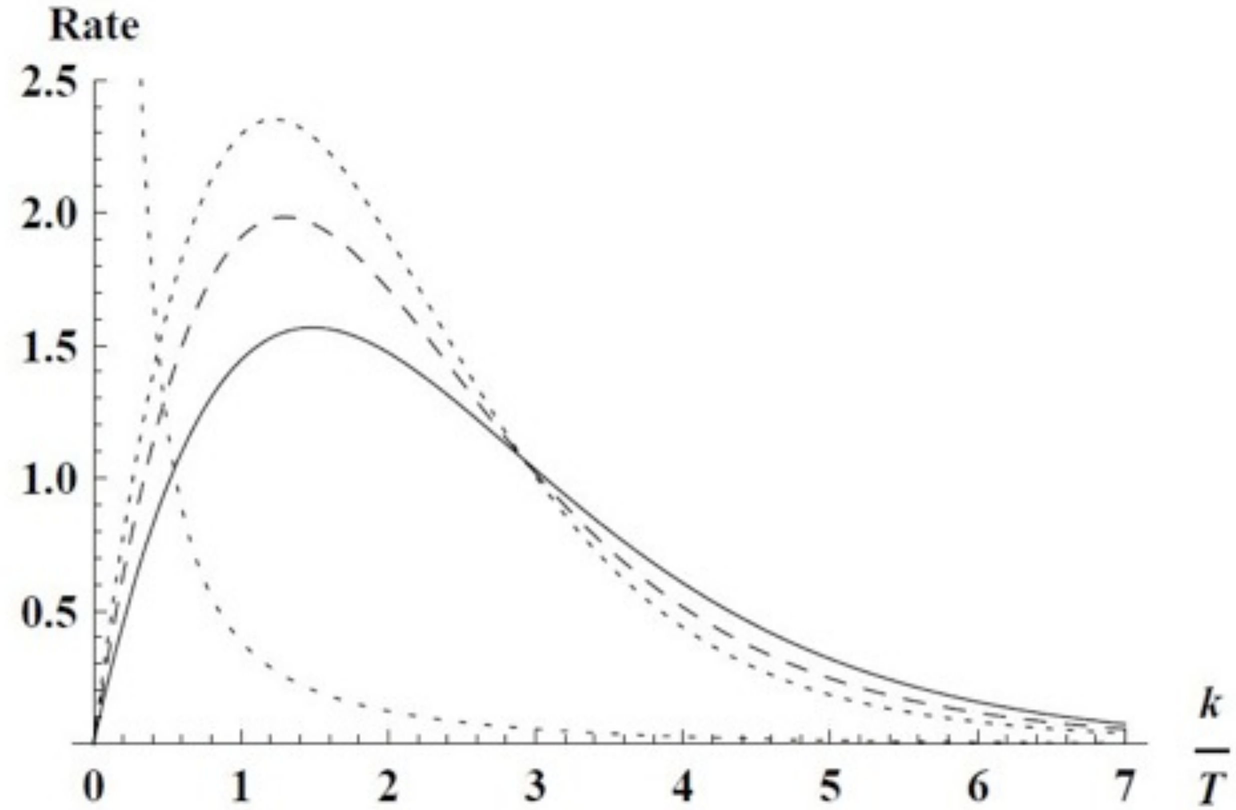
photon production rate for r_s/r_h=1.1, 1.01, 1.001

- enhancement of production rate
- hydro peak broadens and moves right

Photon production rate



photon production rate for r_s/r_h=1.1, 1.01, 1.001



- enhancement of production rate
- hydro peak broadens and moves right
- combining the two allows to study thermalization at finite coupling !

Finite coupling corrections

action: $$S_{IIB} = S^0_{IIB} + S^{\alpha'}_{IIB},$$

$$S^0_{IIB} = \frac{1}{2\kappa_{10}} \int d^{10}x \sqrt{-g} \left(R_{10} - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4.5!}(F_5)^2 \right)$$

$$S^{\alpha'}_{IIB} = \frac{L^6}{2\kappa^2_{10}} \int d^{10}x \sqrt{-g} \left(\gamma e^{\frac{-3}{2}\phi} (C + \mathcal{T})^4 \right), \qquad \gamma \equiv \frac{1}{8}\zeta(3)\lambda^{-\frac{3}{2}}$$

$$\mathcal{T}_{abcdef} = i\nabla_a F^+_{bcdef} + \frac{1}{16}\left(F^+_{abcmn} F^{+\ \ mn}_{def} - 3F^+_{abfmn} F^{+\ \ mn}_{dec} \right),$$

Paulos (2008)

- solving Einsteins equations

$$ds^2 = \frac{r_h^2}{u} \left(-f(u)\, K^2(u)\, dt^2 + d\vec{x}^2 \right) + \frac{1}{4u^2 f(u)} P^2(u)\, du^2 + L^2(u)\, d\Omega_5^2$$

$$K(u) = e^{\gamma\,[a(u)+4b(u)]}, \; P(u) = e^{\gamma\, b(u)}, \; L(u) = e^{\gamma\, c(u)},$$

$$a(u) = -\frac{1625}{8} u^2 - 175\, u^4 + \frac{10005}{16} u^6,$$

$$b(u) = \frac{325}{8} u^2 + \frac{1075}{32} u^4 - \frac{4835}{32} u^6,$$

$$c(u) = \frac{15}{32} (1 + u^2)\, u^4,$$

Gubser et al; Pawelczyk, Theisen (1998)

Finite coupling corrections

equation of motion

- after all the contractions are worked out the eom for a transverse electric field takes the simple form

$$\Psi''(u) - V(u)\Psi(u) = 0$$

Hassanain, Schvellinger

- making the ansatz

$$\Psi_{\substack{\text{in}\\ \text{out}}}(u,) = (1-u)^{\mp\frac{i\hat{\omega}}{2}} \left(\psi^{(0)}_{\substack{\text{in}\\ \text{out}}}(u) + \gamma\, \psi^{(1)}_{\substack{\text{in}\\ \text{out}}}(u) + \mathcal{O}(\gamma^2) \right),$$

- inside solution (pure AdS) stays the same *(Banks, Green (1998))*, but relation between frequencies gets corrected

$$\omega_- = \frac{\omega_+}{\sqrt{f_m}}, \qquad f_m \equiv f(u_s)K^2(u_s)\,,$$

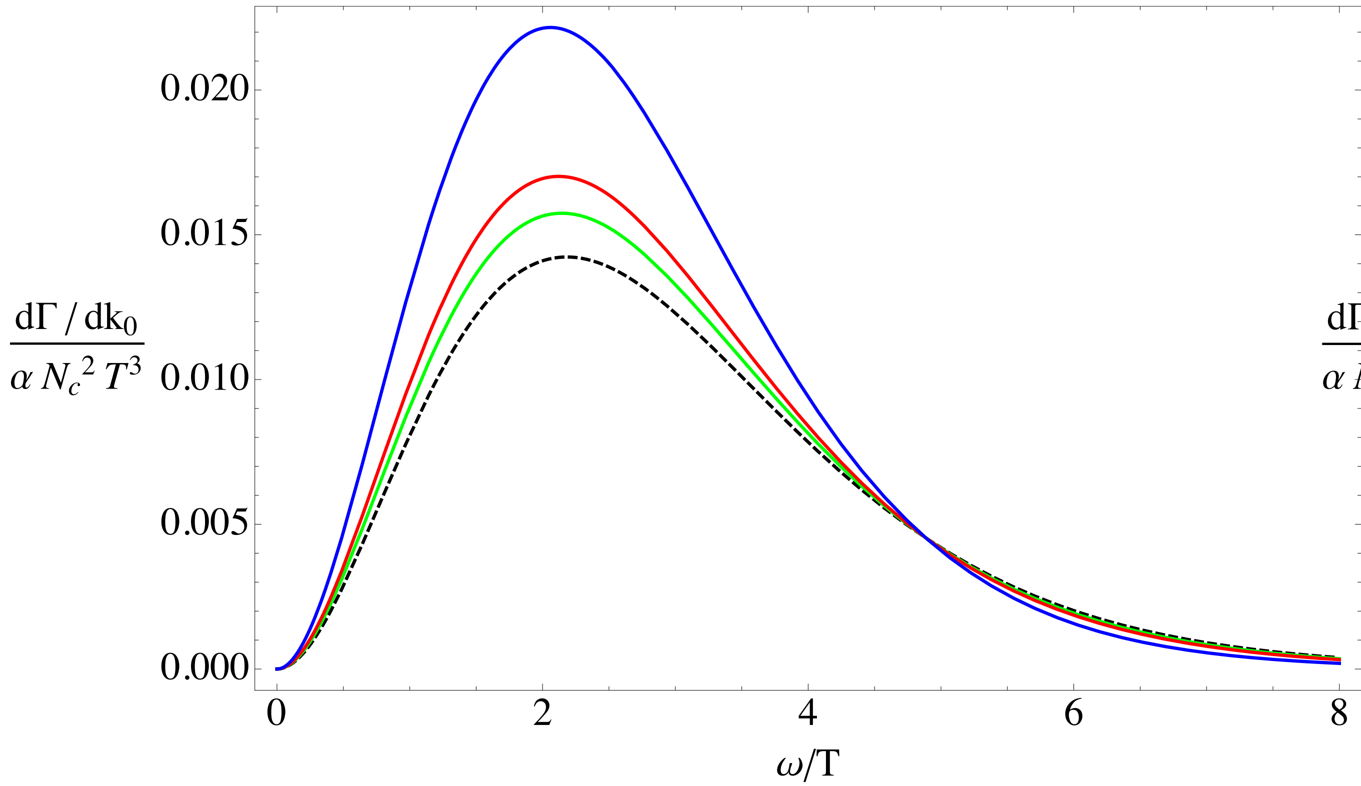
- all the corrections have to be taken into account, e.g

$$\frac{c_-}{c_+} = C_0 + \gamma C_1$$

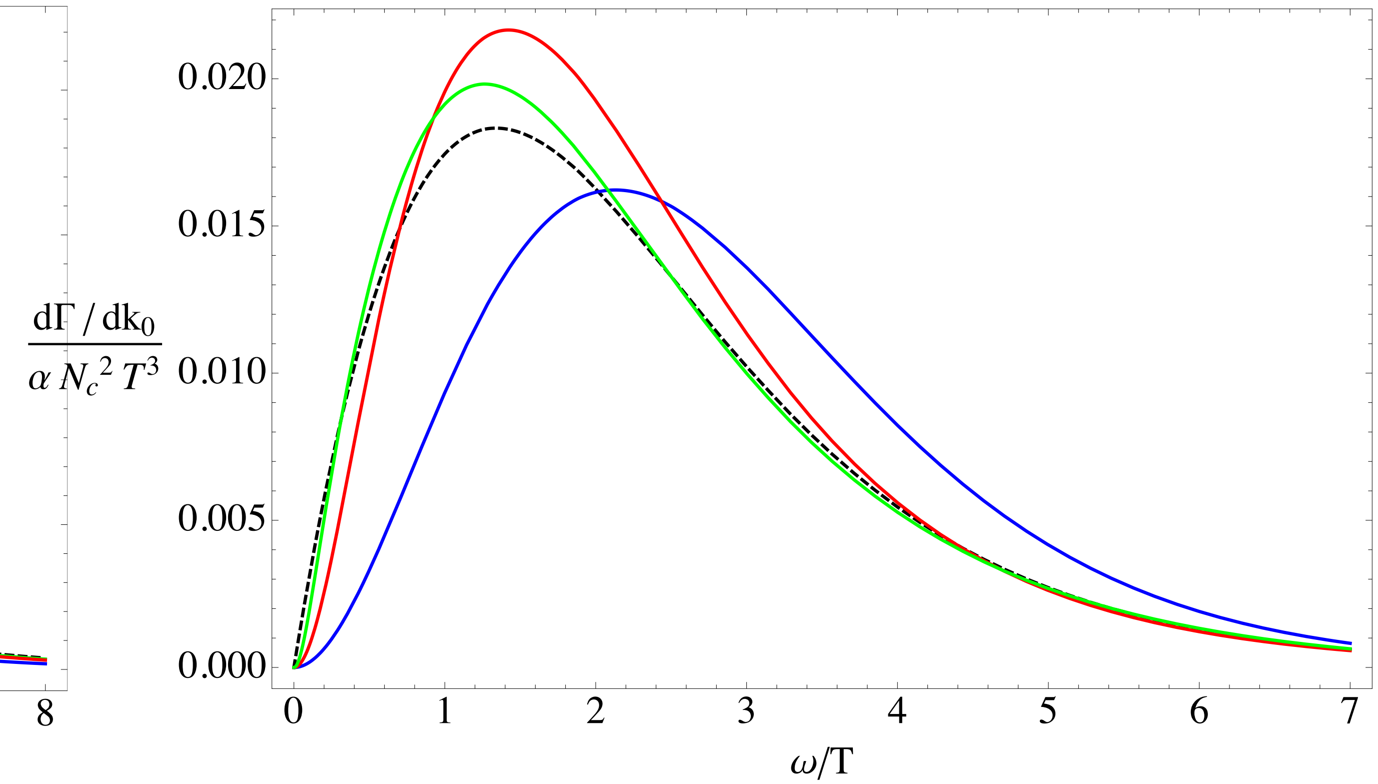
- spectral density

$$\chi(\omega) = \frac{N_c^2 T^2}{2}\left(1 - \frac{265}{8}\gamma\right) \operatorname{Im}\left(\frac{\Psi'_+}{\Psi_+}\right)\Bigg|_{u=0}$$

Photon production rate at finite coupling



emission rate for r_s/r_h=1.01 and $\lambda = \infty,\ 120,\ 80,\ 40$

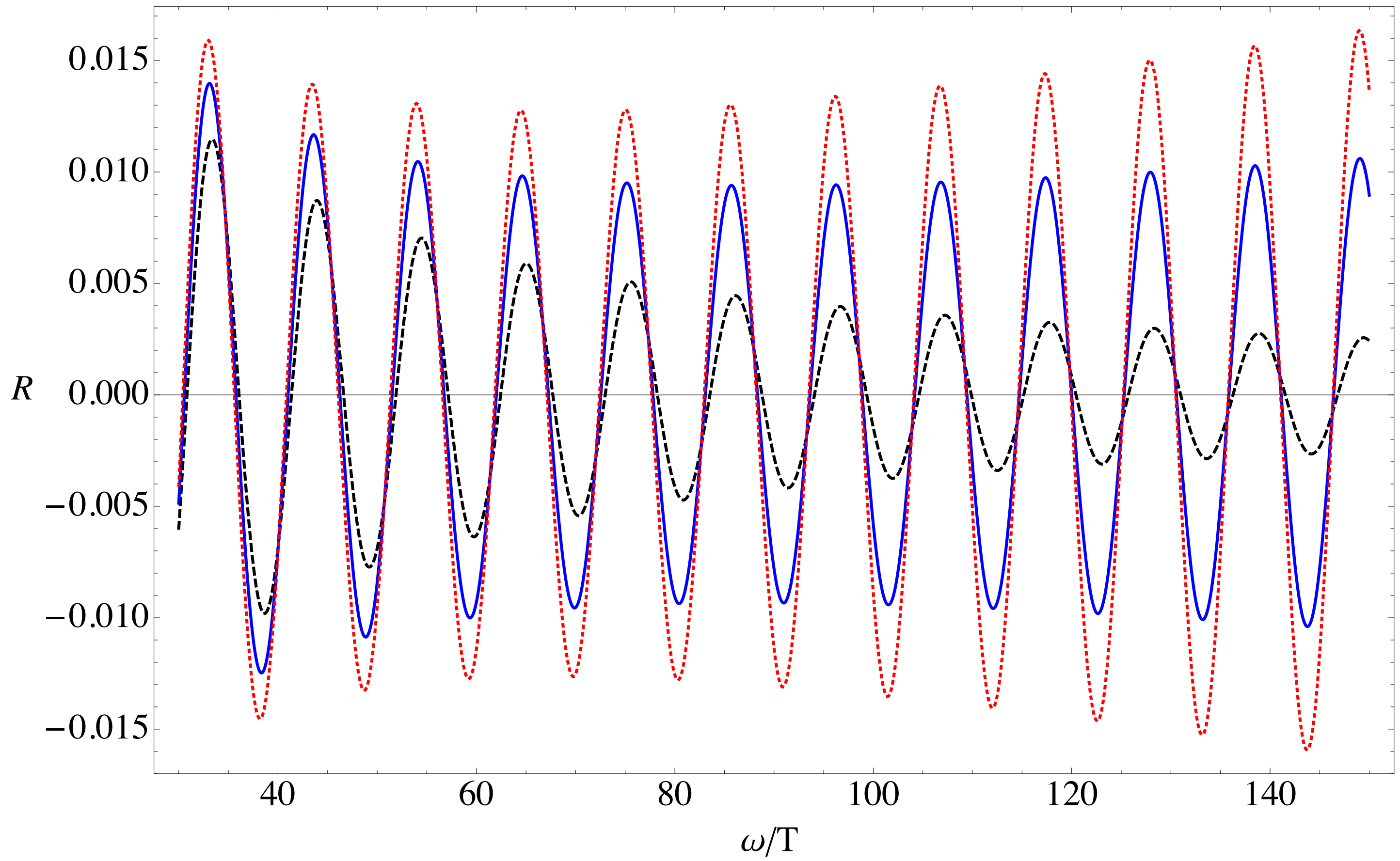


rate for r_s/r_h=1.1, 1.01,1.001 and $\lambda = 100$

- behaviour very similar to thermal limit

Thermalization at finite coupling

relative deviation from thermal limit

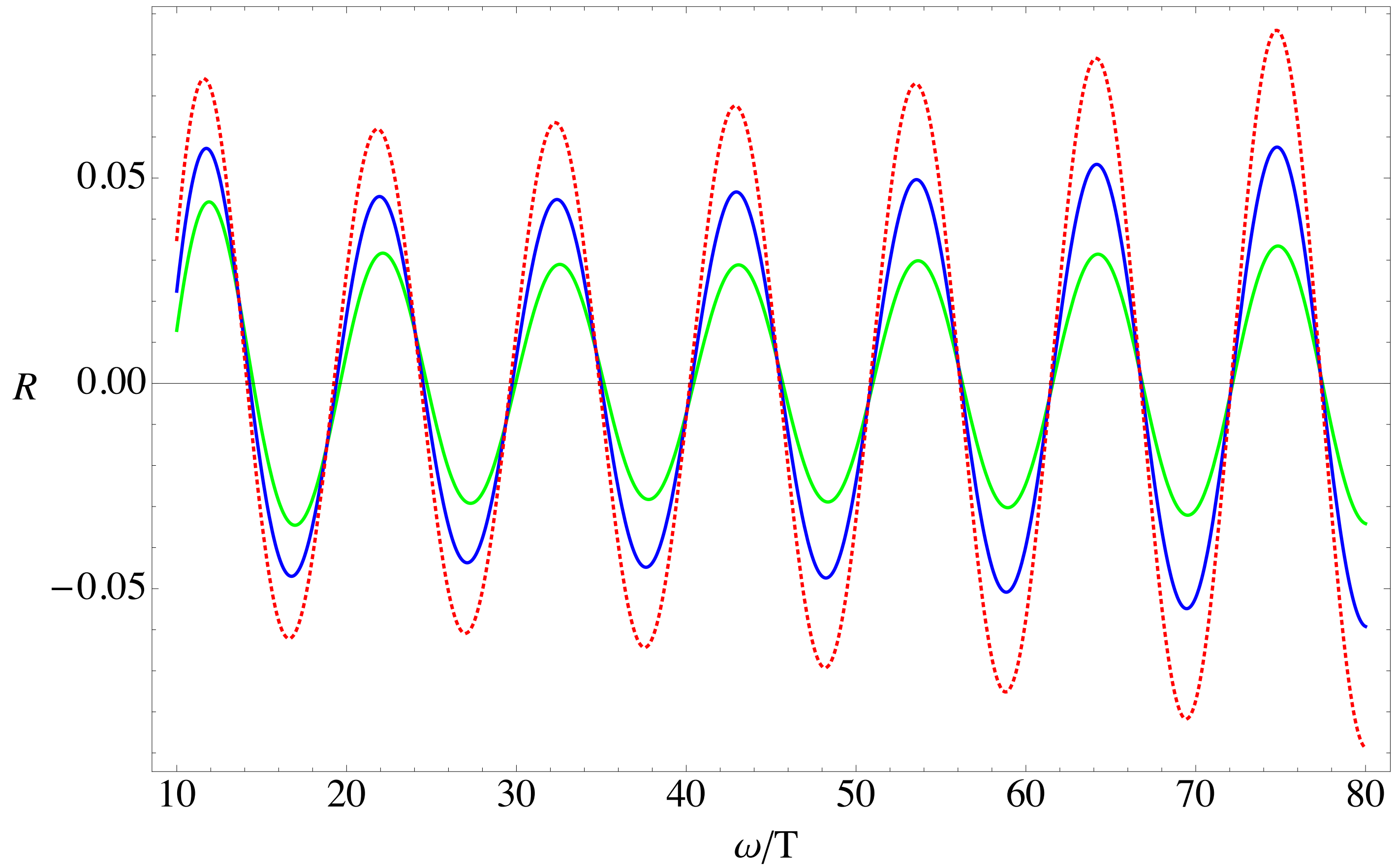


R for r_s/r_h=1.01 and $\lambda = \infty,\ 500,\ 300$

- behaviour of relative deviation changes at large frequency

Thermalization at finite coupling

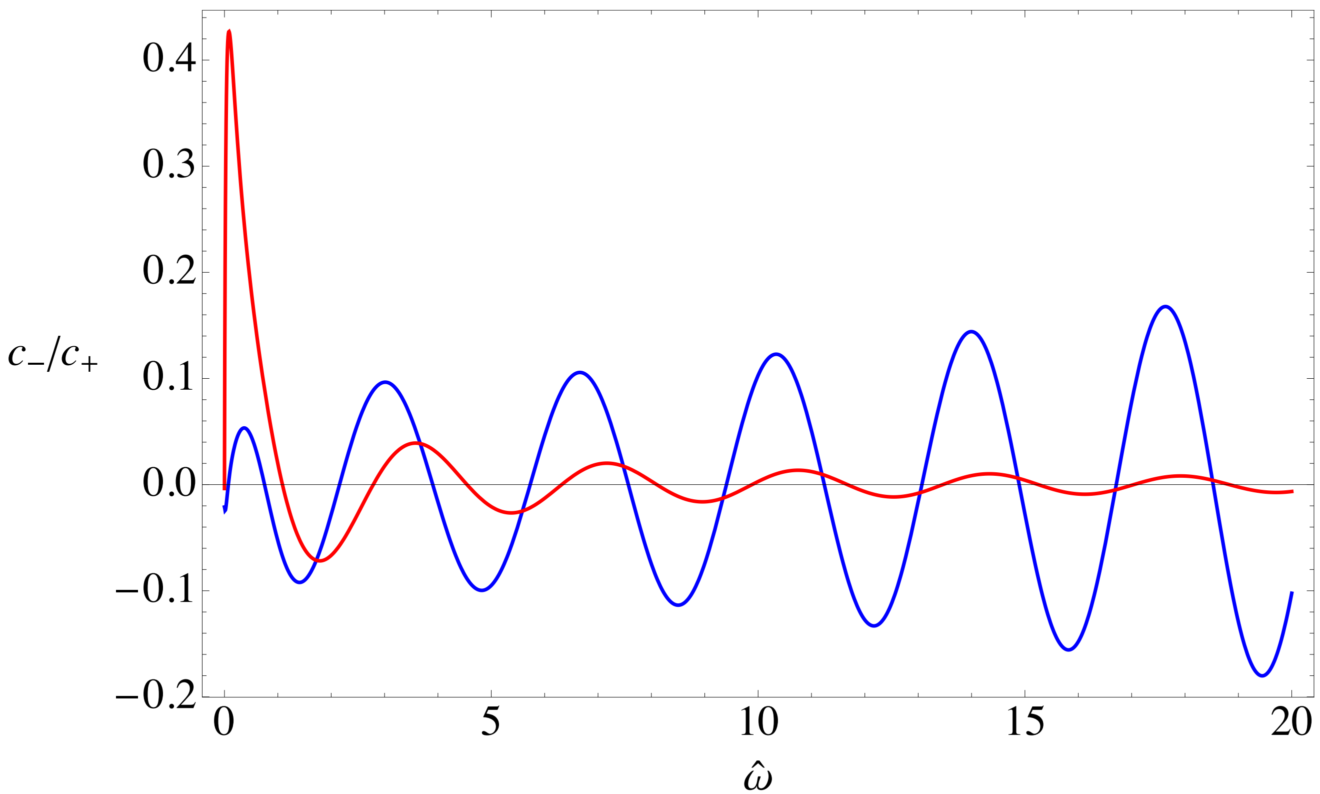
relative deviation from thermal limit



R for r_s/r_h=1.01 and $\lambda = 150,\ 100,\ 75$

- behaviour of relative deviation changes at large frequency
- decreasing the coupling: change happens at lower frequency
- indicates a change of the thermalization pattern from top-down towards bottom-up ?

Thermalization at finite coupling



$$\Pi(\omega) \approx \Pi_{therm} \frac{1 + (C_0 + \gamma C_1)\frac{E'_{out}}{E'_{in}}}{1 + (C_0 + \gamma C_1)\frac{E_{out}}{E_{in}}}$$

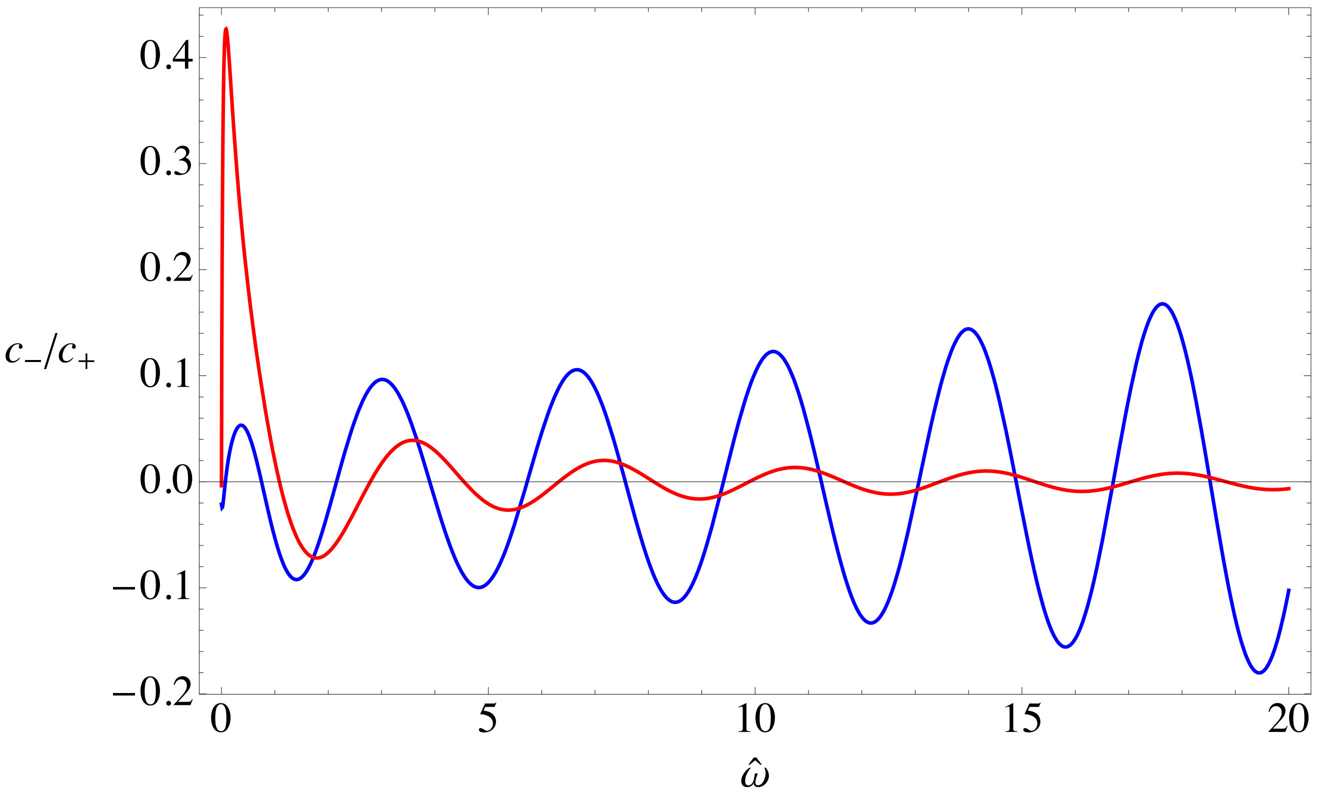
$$C_0 \approx \frac{1}{\omega}, \qquad C_1 \approx \omega$$

- behaviour of the fields near the horizon is crucial
- originates from the Schroedinger potential

WKB approximation

$$\chi(\hat{\omega}) \approx \hat{\omega}^{\frac{2}{3}} \left(1 + \frac{3\zeta(3)}{8\lambda^{\frac{3}{2}}} + \frac{f_1(u_s)}{\hat{\omega}} + \frac{f_2(u_s)\hat{\omega}}{\lambda^{\frac{3}{2}}}\right)$$

Thermalization at finite coupling



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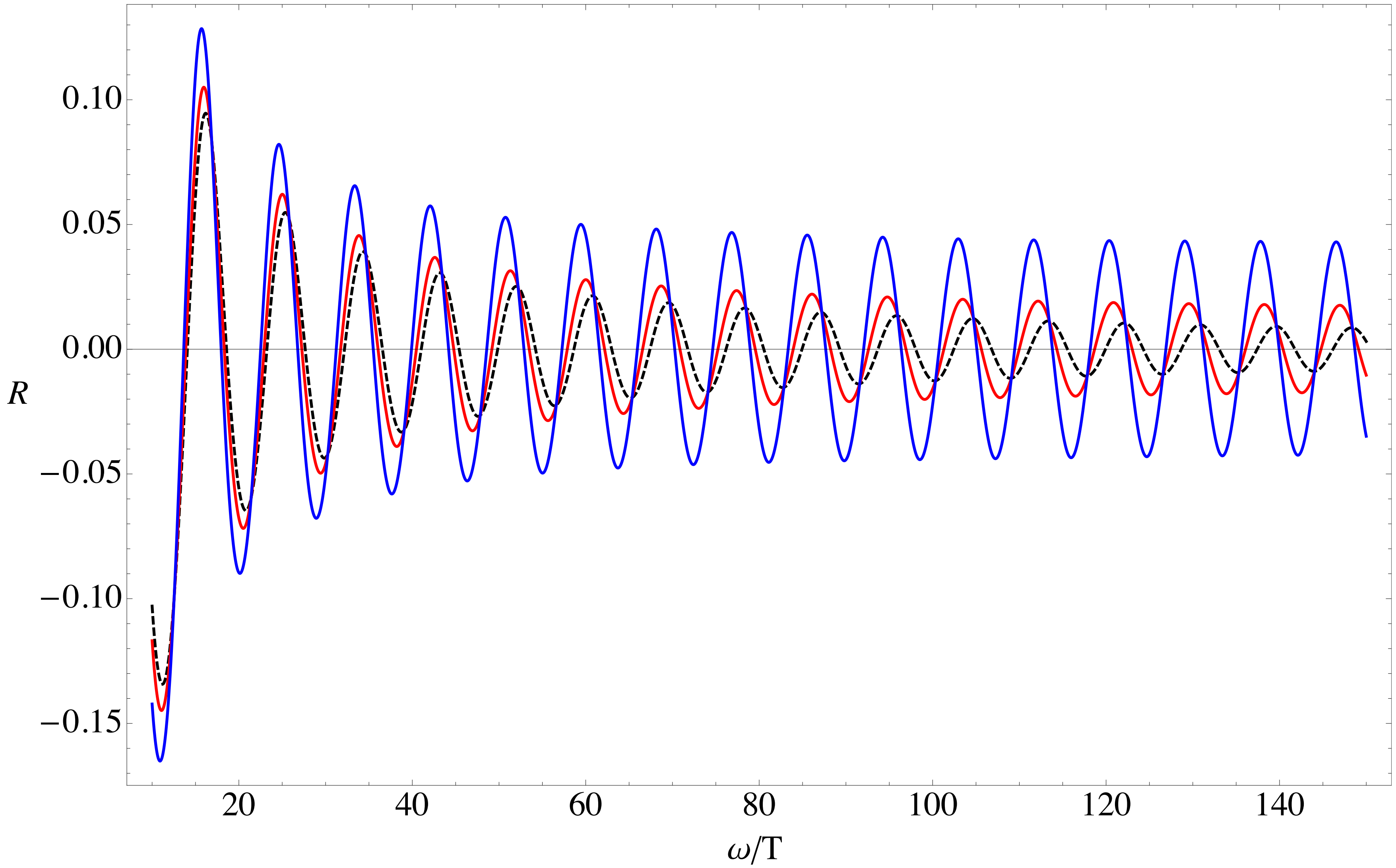
$$\chi(\hat{\omega}) \approx \hat{\omega}^{\frac{2}{3}} \left(1 + \frac{3\zeta(3)}{8\lambda^{\frac{3}{2}}} + \frac{f_1(u_s)}{\hat{\omega}} + \frac{f_2(u_s)\hat{\omega}}{\lambda^{\frac{3}{2}}} \right)$$

- so far: only photons that get emitted from the plasma
- what about plasma constituents themselves ?

Future directions I: $\langle T_{\mu\nu} T_{\alpha\beta} \rangle$

Relative deviation of the shear channel: $\langle T_{xy} T_{xy} \rangle$

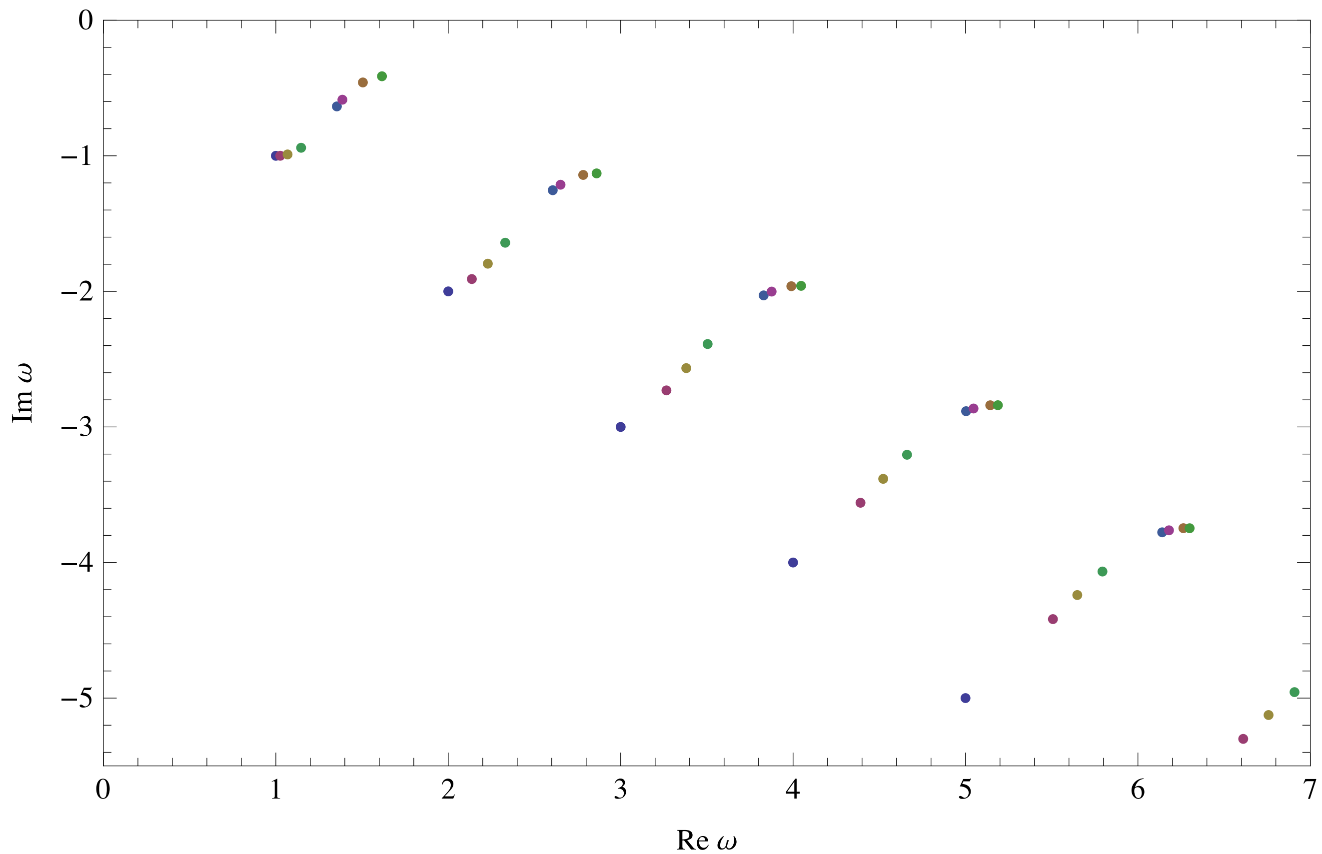


relative deviation R for r_s/r_h=1.1 and $\lambda = \infty,\ 100,\ 50$

- finite coupling effects are weaker
- for large energies relative deviation becomes constant
- can be seen from the behaviour of c_-/c_+

Future directions II: QNM analysis

QNM for R current correlator at infinite coupling



- flow of the imaginary part of the first QNM:

$$\mathrm{Im}\,\omega_1 = 2\pi T\left(-1+\frac{c}{\lambda^{\frac{3}{2}}}\right)$$

Conclusion

thermalization at infinite coupling

- enhancement of production rate
- top down thermalization
- depends on virtuality: on-shell photons are last to thermalize

thermalization at finite coupling

- enhancement of production rate
- indication of thermalization pattern changing from top down towards bottom up

open questions

- why does the causality argument not apply
- go beyond quasistatic approximation
- can one include finite coupling corrections in more involved models of holographic thermalization