Conformal Regge Theory

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Work with J. Penedones and V. Gonçalves, 1209.4355 [hep-th]

HoloGrav

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- Regge theory gives important physical information in QCD

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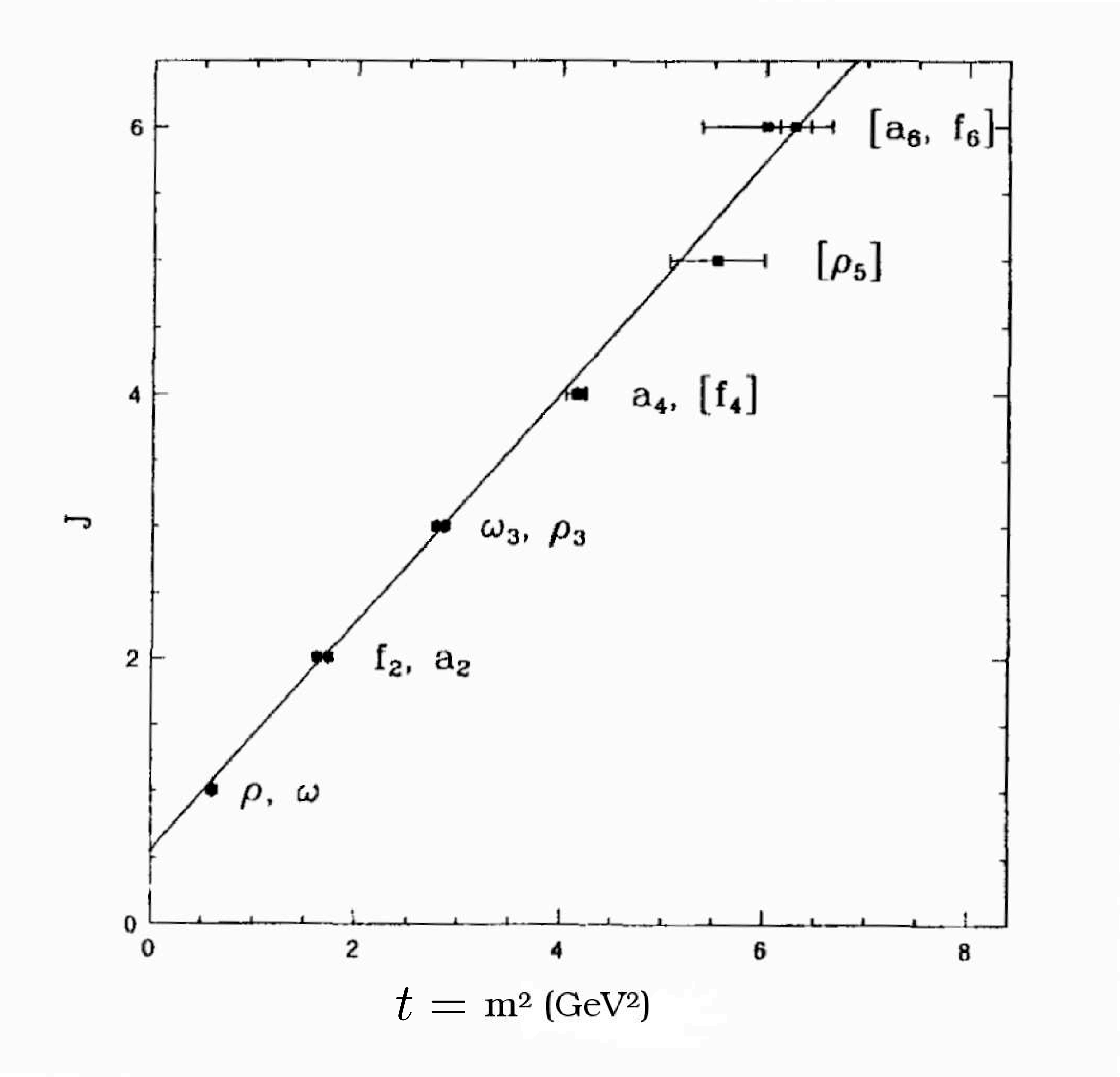
Regge trajectory for isospin $I = 1$ even parity mesons.



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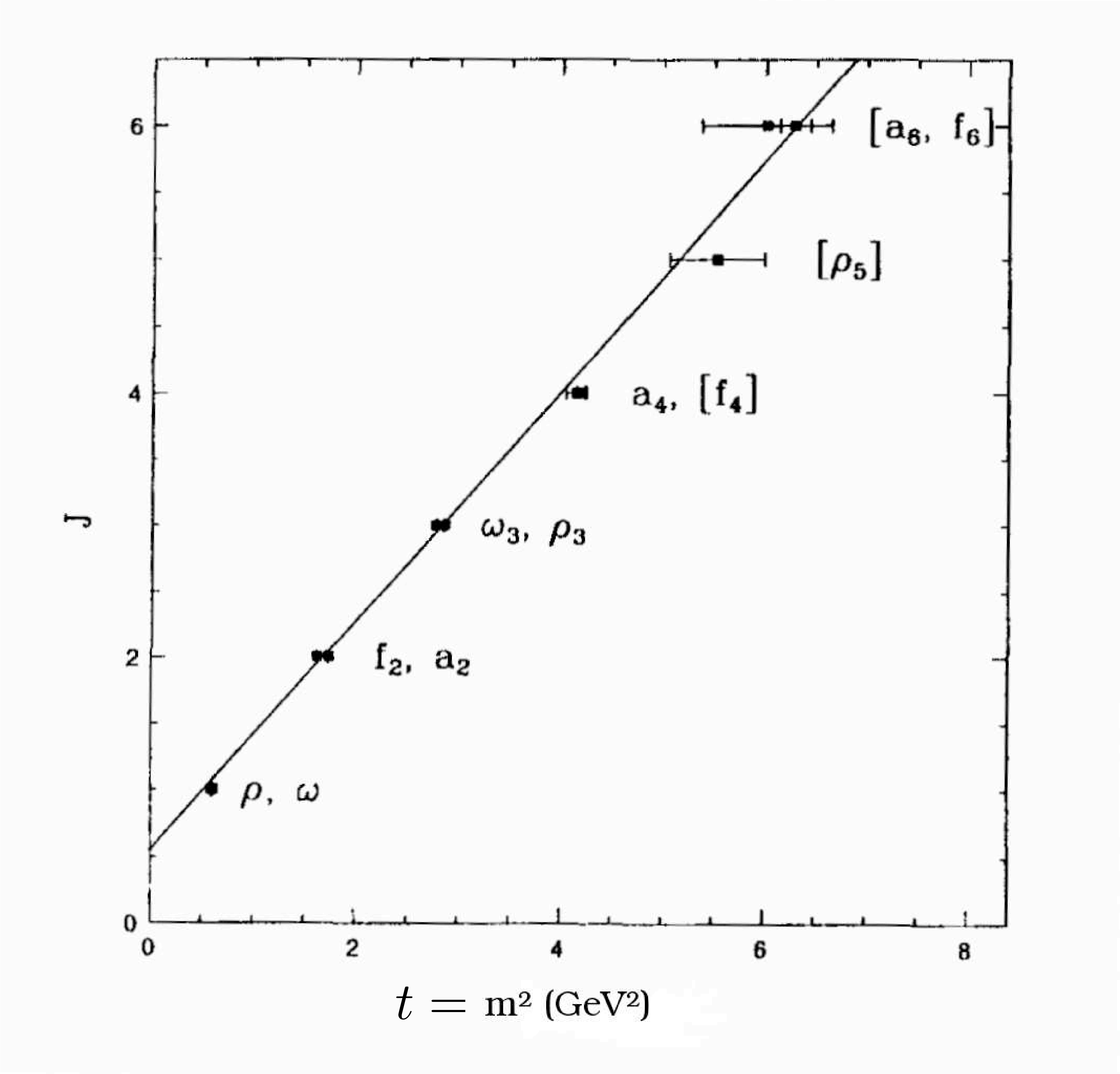
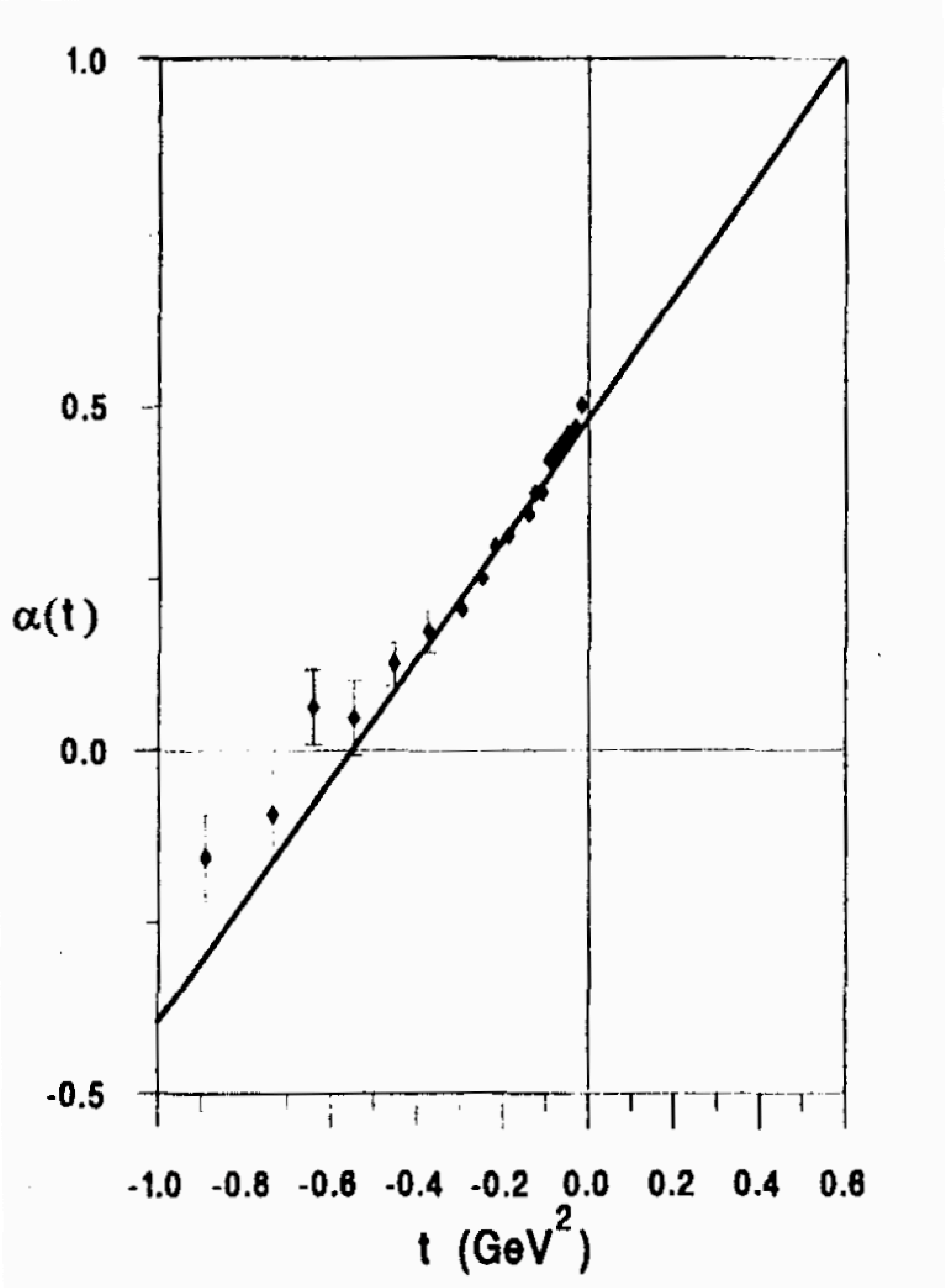


These mesons dominate exchange in $\pi^- + p \to \pi^0 + n$



$s \gg -t$

$$A(s,t) \sim \beta(t)\, s^{j(t)}$$

Total cross section

$$\sigma \sim s^{j(0)-1}$$

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Regge theory in CFT's?

[Cornalba 07;
Cornalba, MSC, Penedones 08]

AdS/CFT duality

Strings in AdS (d+1 dimensions) ⟷ Conformal Field Theory (d dimensions)

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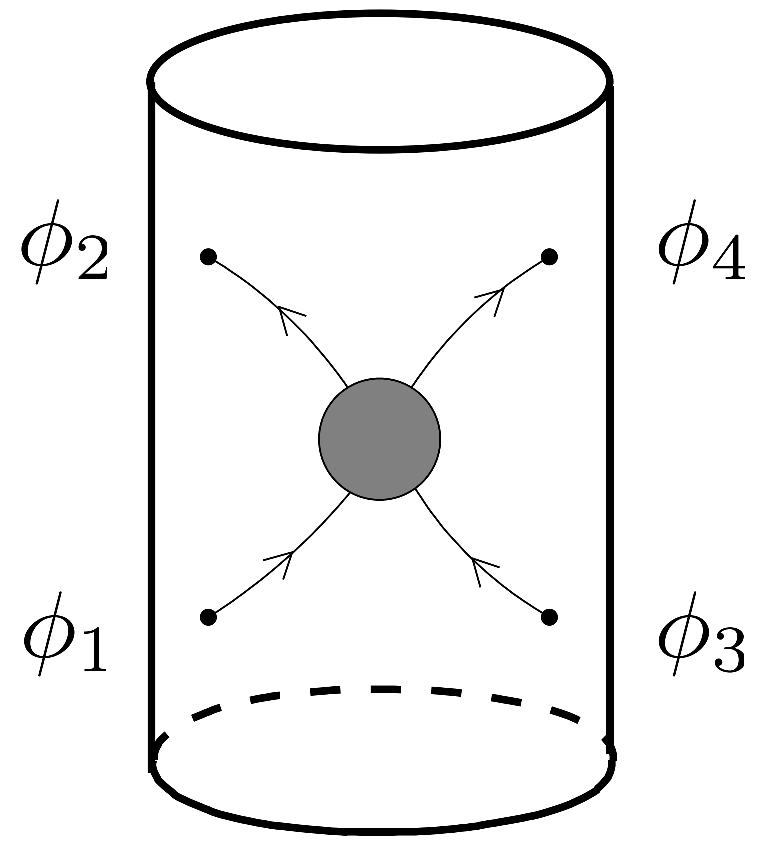
Tree level $g_s \to 0$	Planar level $N \to \infty$
Finite string length $l_s = \sqrt{\alpha'}$	Finite ‘t Hooft coupling $\lambda = g_{YM}^2 N = \dfrac{R^4}{\alpha'^2}$
String fields ϕ	Single trace operators $\mathcal{O}$



$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$$

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- Extract non-trivial and new information about anomalous dimensions [Kotikov, Lipatov, Staudacher,Velizhanin 07], graviton Regge trajectory in AdS and some OPE coefficients in N=4 Super Yang-Mills (today)

Regge theory in String Theory

- Virasoro-Shapiro S-matrix element

$$\mathcal{T}(s,t) = 8\pi G_N \left(\frac{tu}{s} + \frac{su}{t} + \frac{st}{u}\right) \frac{\Gamma\left(1 - \frac{\alpha' s}{4}\right)\Gamma\left(1 - \frac{\alpha' u}{4}\right)\Gamma\left(1 - \frac{\alpha' t}{4}\right)}{\Gamma\left(1 + \frac{\alpha' s}{4}\right)\Gamma\left(1 + \frac{\alpha' u}{4}\right)\Gamma\left(1 + \frac{\alpha' t}{4}\right)}$$

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- Regge limit

$$s \gg -t \qquad \approx \underbrace{\frac{32\pi G_N}{\alpha'} e^{-\frac{i\pi\alpha' t}{4}} \frac{\Gamma\left(-\frac{\alpha' t}{4}\right)}{\Gamma\left(1+\frac{\alpha' t}{4}\right)}}_{\beta(t)} \left(\frac{\alpha' s}{4}\right)^{\underbrace{2+\frac{\alpha' t}{2}}_{j(t)}}$$

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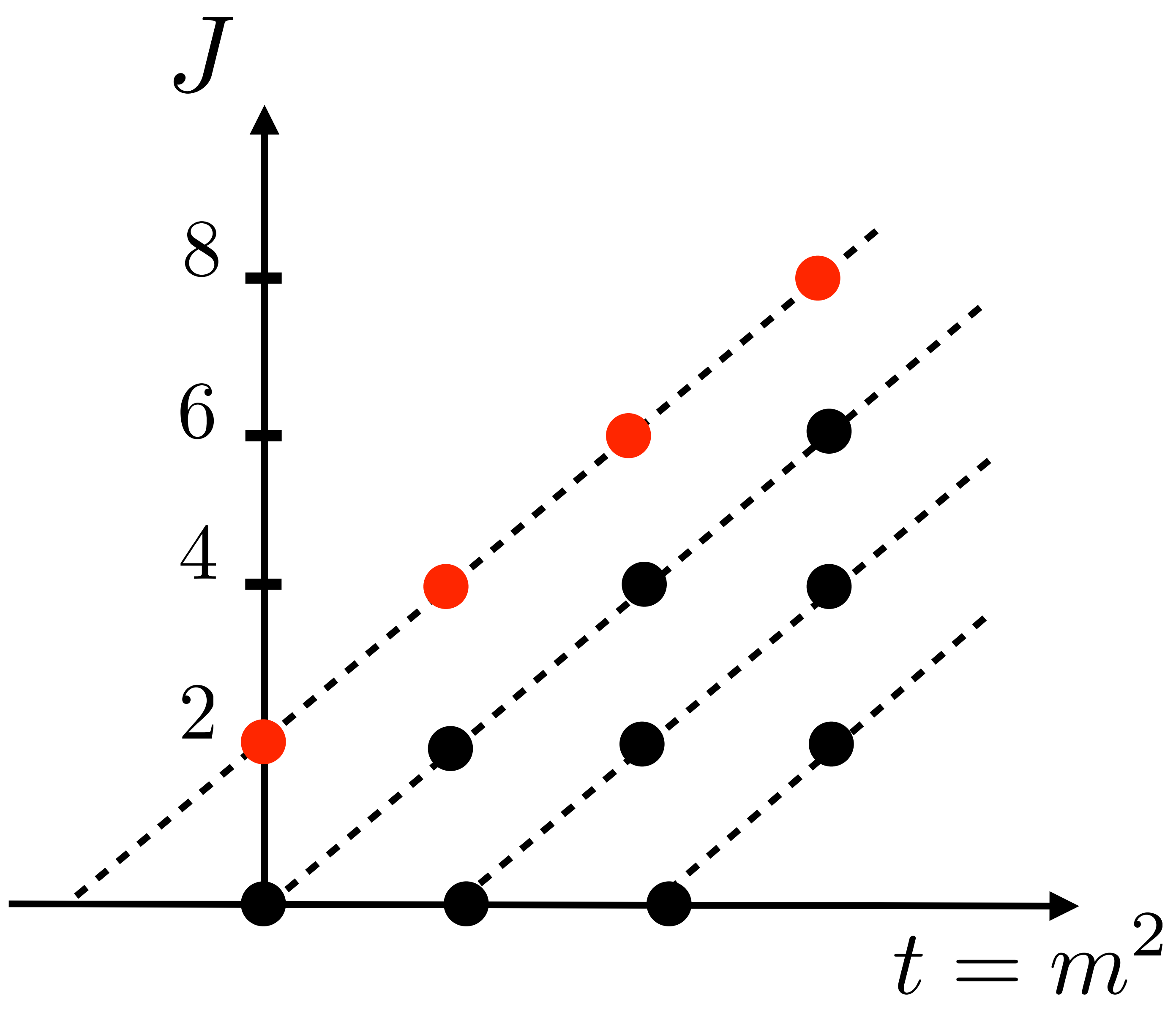
- Amplitude contains poles for each physical exchange. The Regge behaviour can be obtained only from exchange of particles in leading Regge trajectory



- t-channel partial wave expansion

$$\mathcal{T}(s,t)=\sum_{J=0}^{\infty} a_J(t)\, P_J\left(1+2\frac{s}{t}\right) \longrightarrow \sim \left(\frac{s}{t}\right)^J$$

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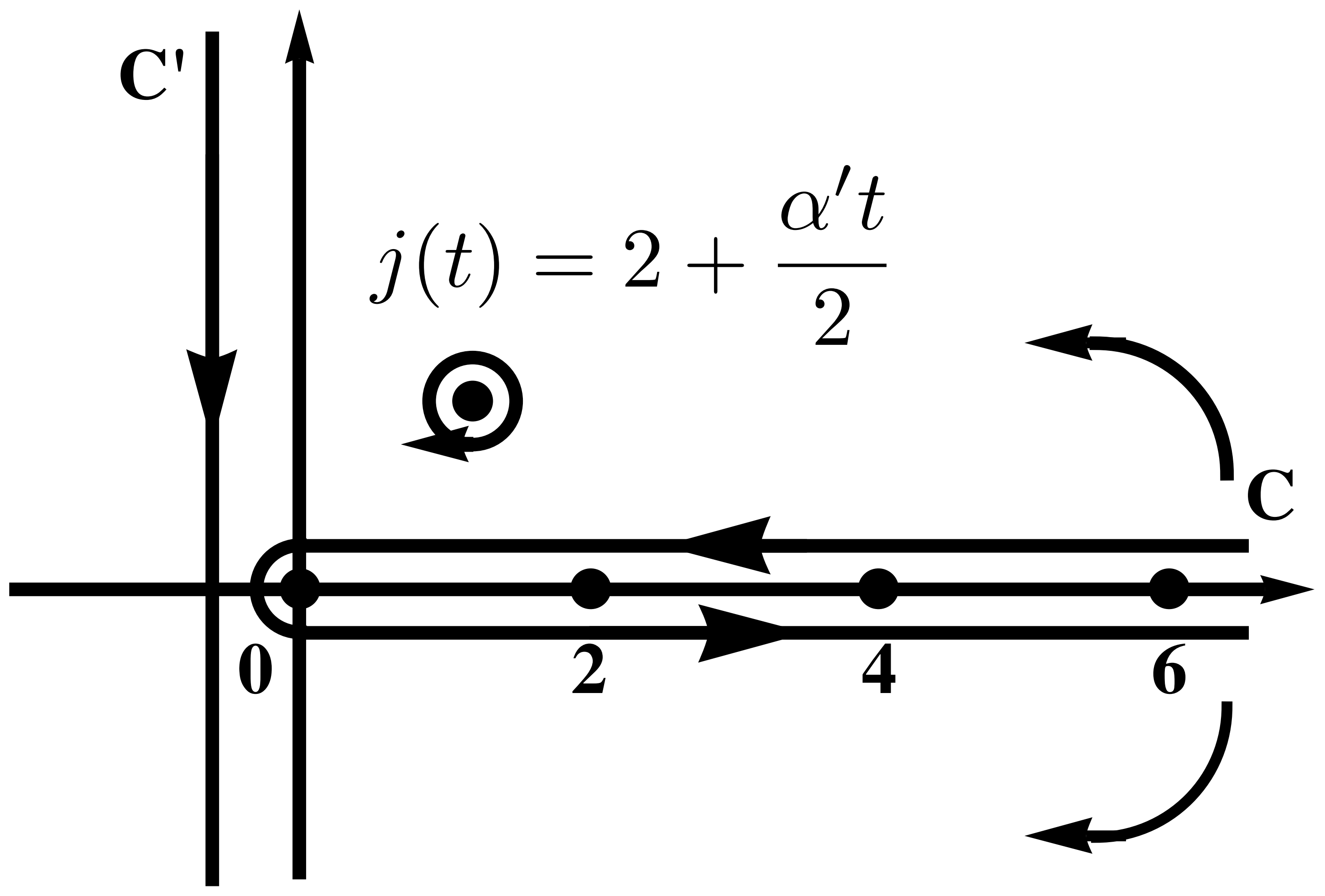
$$\sum_J \to \int_C \frac{dJ}{2\pi i} \frac{\pi}{\sin(\pi J)}$$

- Analytically continue in J and pick leading pole from

$$\boxed{\mathcal{T}(s,t) \approx \beta(t) s^{j(t)}}$$

$$a_J(t) \approx -\frac{j'(t)\, r(j(t))}{J-j(t)}$$



Resume & what's next

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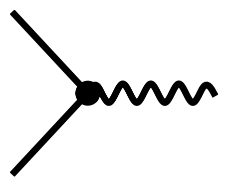
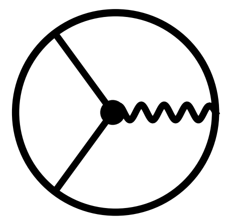
Cubic couplings $C(J)\sim$ 	3-pt functions or OPE coefficients $C(J)\sim$ 
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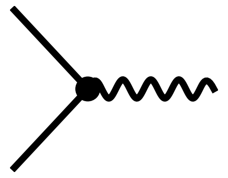
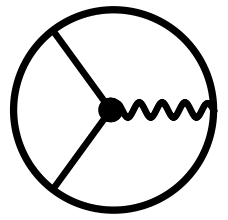
Cubic couplings $C(J)\sim$ 	3-pt functions or OPE coefficients $C(J)\sim$ 
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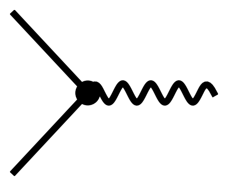
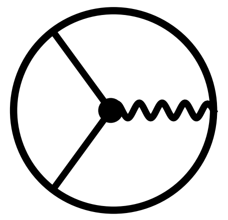
Cubic couplings $C(J) \sim$ 	3-pt functions or OPE coefficients $C(J) \sim$ 
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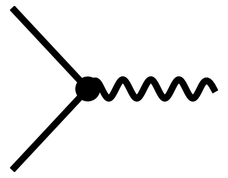
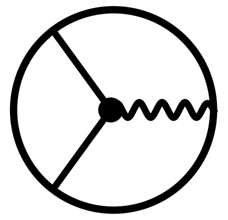
Cubic couplings $C(J)\sim$ 	3-pt functions or OPE coefficients $C(J)\sim$ 
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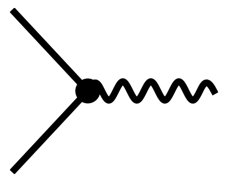
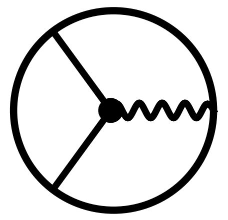
Cubic couplings $C(J)\sim$ 	3-pt functions or OPE coefficients $C(J)\sim$ 
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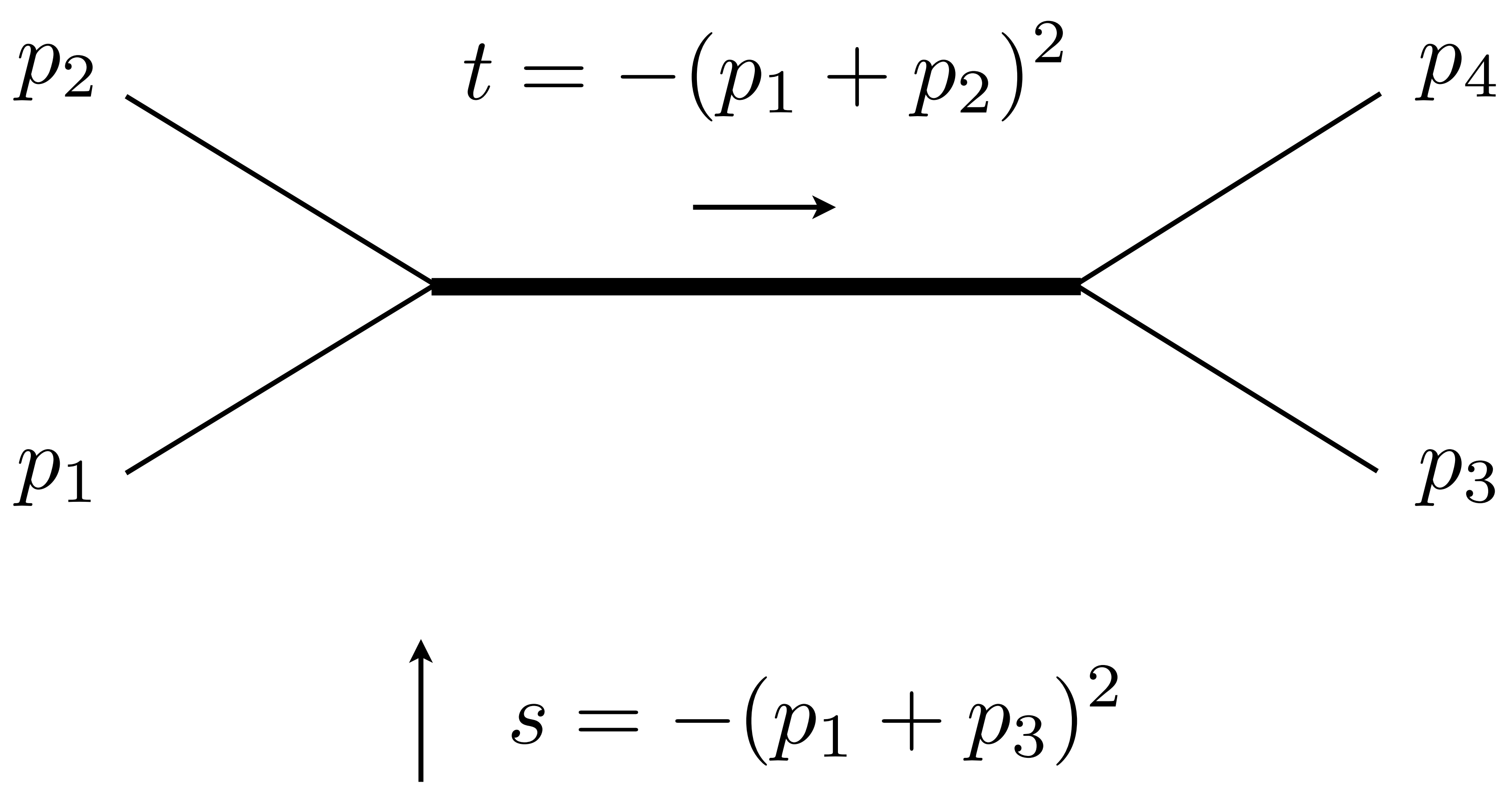

$$\tau(s,t) =$$
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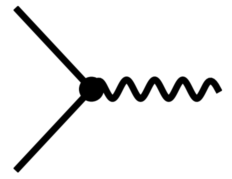
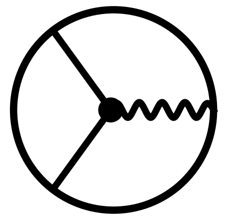
Cubic couplings $C(J) \sim$ 	3-pt functions or OPE coefficients $C(J) \sim$ 
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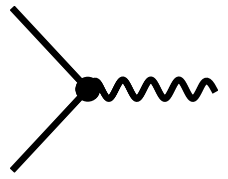
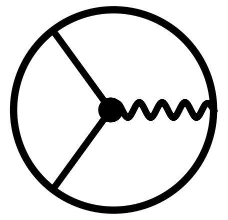
Cubic couplings $C(J)\sim$ 	3-pt functions or OPE coefficients $C(J)\sim$ 
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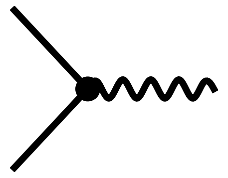
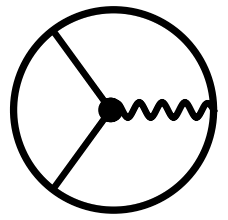
Cubic couplings $C(J) \sim$ 	3-pt functions or OPE coefficients $C(J) \sim$ 
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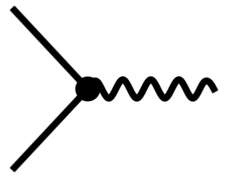
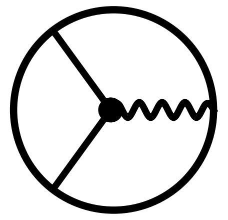
Cubic couplings $C(J) \sim$ 	3-pt functions or OPE coefficients $C(J) \sim$ 
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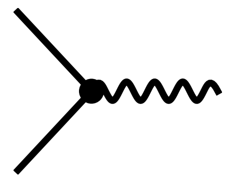
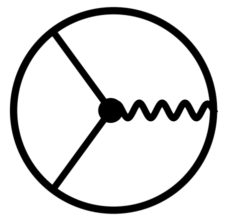
Cubic couplings $C(J) \sim$ 	3-pt functions or OPE coefficients $C(J) \sim$ 
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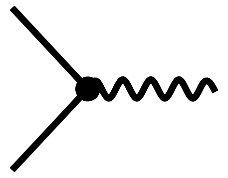
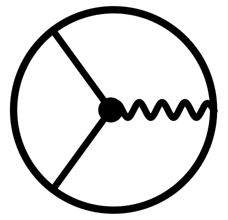
Cubic couplings $C(J) \sim$ 	3-pt functions or OPE coefficients $C(J) \sim$ 
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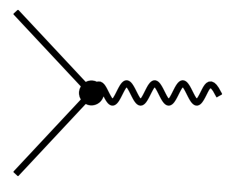
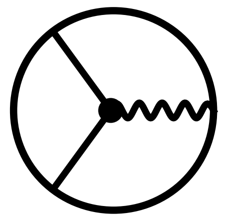
Cubic couplings $C(J)\sim$ 	3-pt functions or OPE coefficients $C(J)\sim$ 
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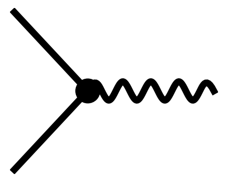
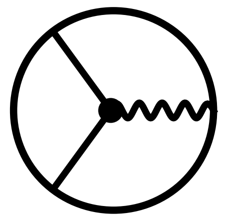
Cubic couplings $C(J) \sim$ 	3-pt functions or OPE coefficients $C(J) \sim$ 
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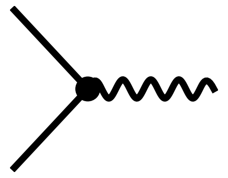
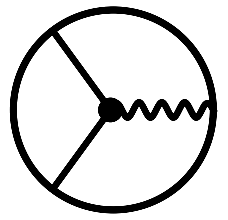
Cubic couplings $C(J) \sim$ 	3-pt functions or OPE coefficients $C(J) \sim$ 
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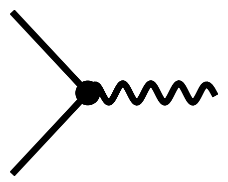
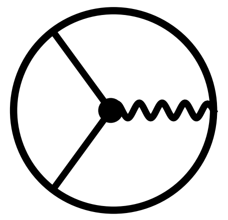
Cubic couplings $C(J)\sim$ 	3-pt functions or OPE coefficients $C(J)\sim$ 
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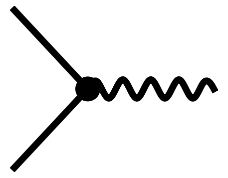
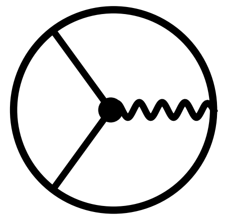
Cubic couplings $C(J)\sim$ 	3-pt functions or OPE coefficients $C(J)\sim$ 
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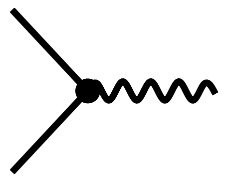
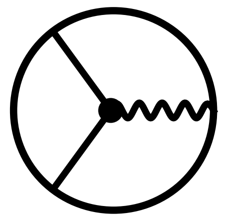
Cubic couplings $C(J)\sim$ 	3-pt functions or OPE coefficients $C(J)\sim$ 
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Conformal symmetry constrains variables

$$\sum_{j=1}^{4} \delta_{ij} = 0 \qquad (\delta_{ii} = -\Delta_i)$$

Introduce fictitious momenta such that

$$\sum_{i=1}^{4} p_i = 0 \ , \qquad \delta_{ij} = p_i \cdot p_j$$

“Maldelstam” variables

$$s = -(p_1 + p_3)^2 - \Delta_1 - \Delta_4 = \Delta_3 - \Delta_4 - 2\delta_{13}$$

$$t = -(p_1 + p_2)^2 = \Delta_1 + \Delta_2 - 2\delta_{12}$$

Mellin amplitude - definition [Mack 09; Penedones 10]

- Correlators can be thought as S-matrix elements for AdS scattering. Mellin amplitudes make analogy more explicit (can write Feynman rules)

$$A(x_i) = \langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_4(x_4) \rangle_{\mathrm{c}} = \int [d\delta] M(\delta_{ij}) \prod_{1 \le i < j \le 4} \Gamma(\delta_{ij}) (x_{ij}^2)^{-\delta_{ij}}$$

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cross ratios

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \,, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

$$\boxed{\mathcal{A}(u,v) = \int_{-i\infty}^{i\infty} \frac{dt ds}{(4\pi i)^2} M(s,t) \, u^{t/2} v^{-(s+t)/2}} \times \text{product of } \Gamma \text{ functions}$$

Mellin amplitude - OPE [Mack 09; Penedones 10]

$$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \frac{C_{12k}}{(x^2)^{\frac{1}{2}(\Delta_1+\Delta_2-\Delta_k)}} \left[\frac{x_{\mu_1}\dots x_{\mu_{J_k}}}{(x^2)^{\frac{J_k}{2}}} \mathcal{O}_k^{\mu_1\dots\mu_{J_k}}(0) + \text{descendants} \right]$$

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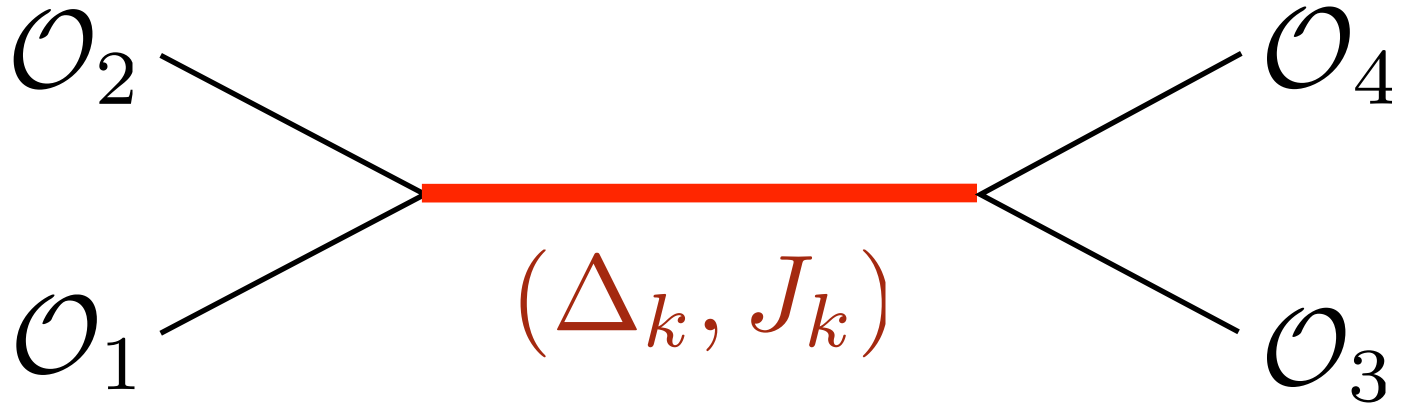
- Conformal block expansion of reduced correlator



$$\mathcal{A}(u,v)=\sum_k C_{12k}C_{34k}\,G_{\Delta_k,J_k}(u,v)$$

$$\approx\sum_k C_{12k}C_{34k}\,\frac{J_k!}{2^{J_k}\,(h-1)_{J_k}}\,u^{\frac{\Delta_k}{2}}\,C_{J_k}^{h-1}\left(\frac{v-1}{2\sqrt{u}}\right)$$

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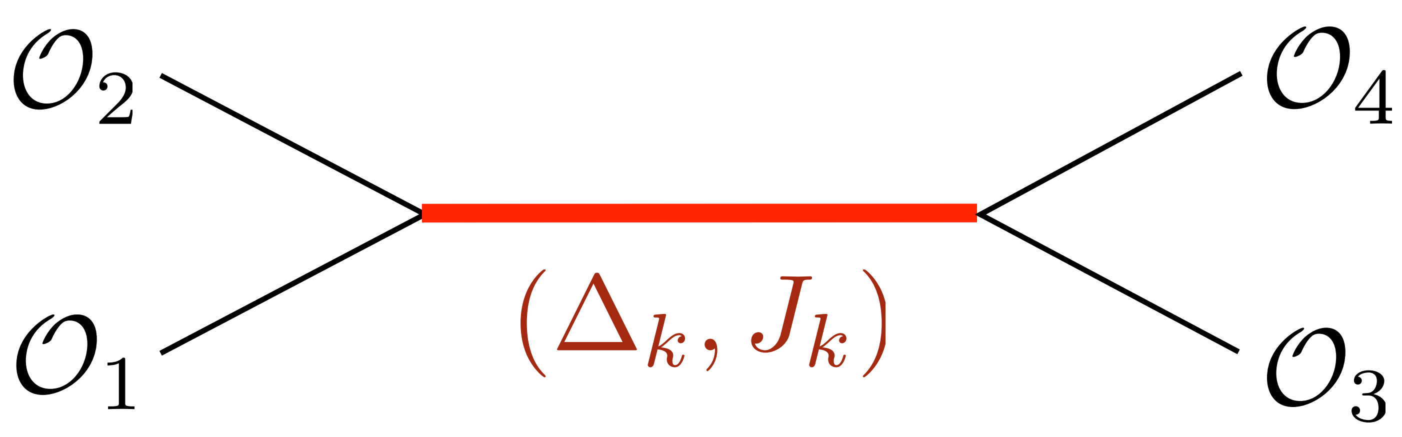

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- In Mellin space must have poles in t variable

$$M(s,t)\approx\frac{C_{12k}C_{34k}\,Q_{J,m}(s)}{t-\Delta+J-2m}\,,\qquad m=0,1,2,\dots$$

Mellin amplitude - conformal partial wave expansion

- In Mellin space write partial wave expansion

$$M(s,t) = \sum_{J=0}^{\infty} \int_{-\infty}^{\infty} d\nu \, b_J(\nu^2) \, M_{\nu,J}(s,t)$$

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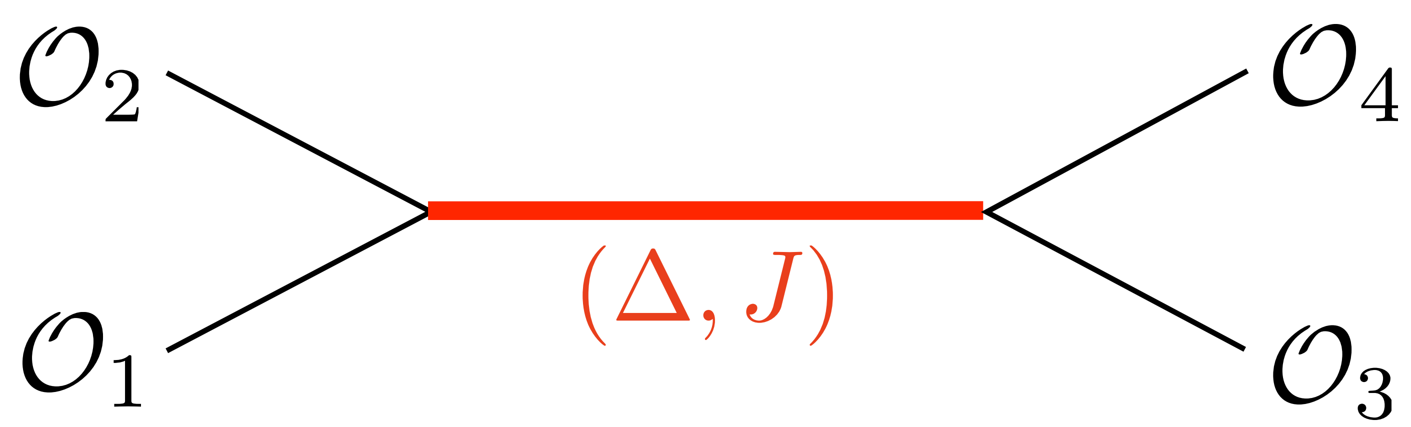

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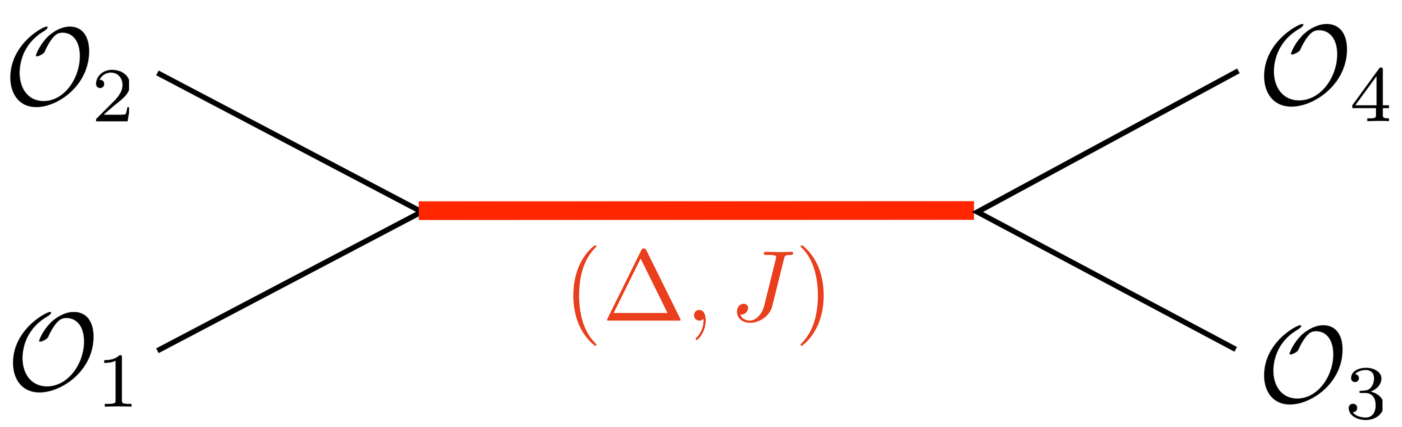

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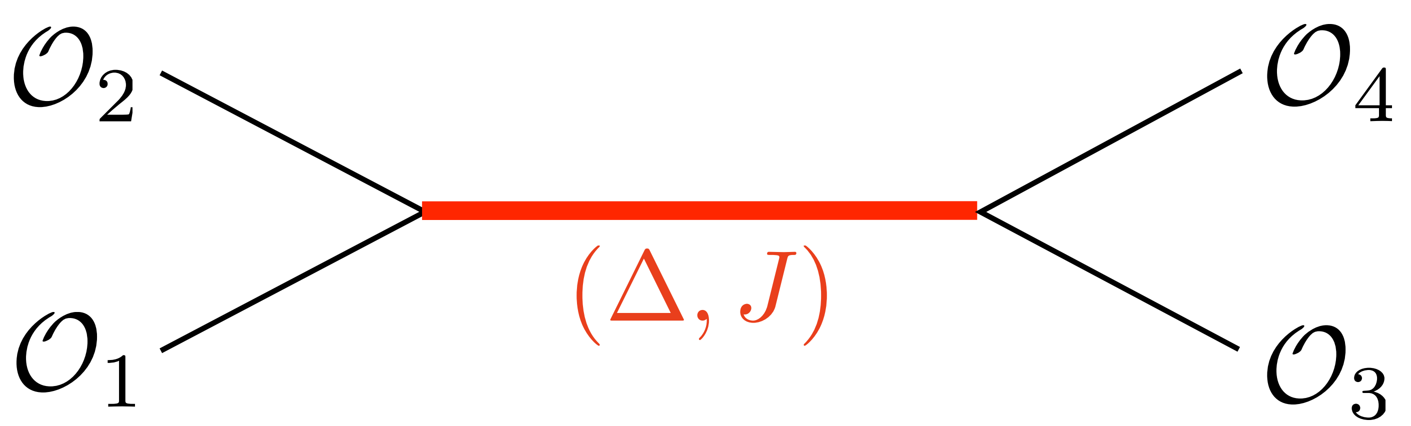

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- Flat space limit $\qquad M(s,t) \;\rightarrow\; \mathcal{T}(s,t) = \sum_{J=0}^{\infty} a_J(t) \, P_J \left(1 + 2\frac{s}{t}\right)$

Conformal Regge theory

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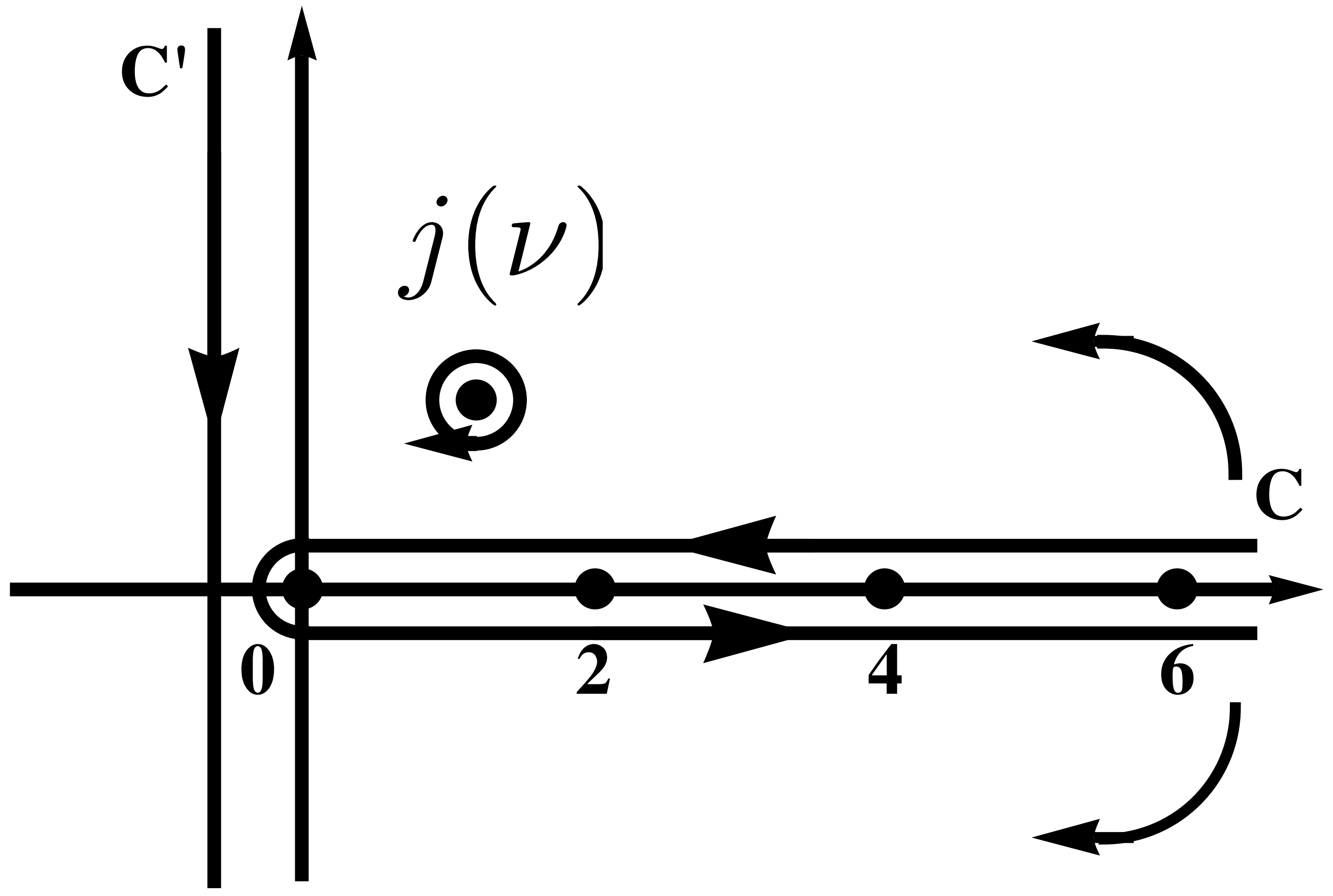
$$\sum_J \to \int_{\mathrm{C}} \frac{dJ}{2\pi i} \frac{\pi}{\sin(\pi J)}$$



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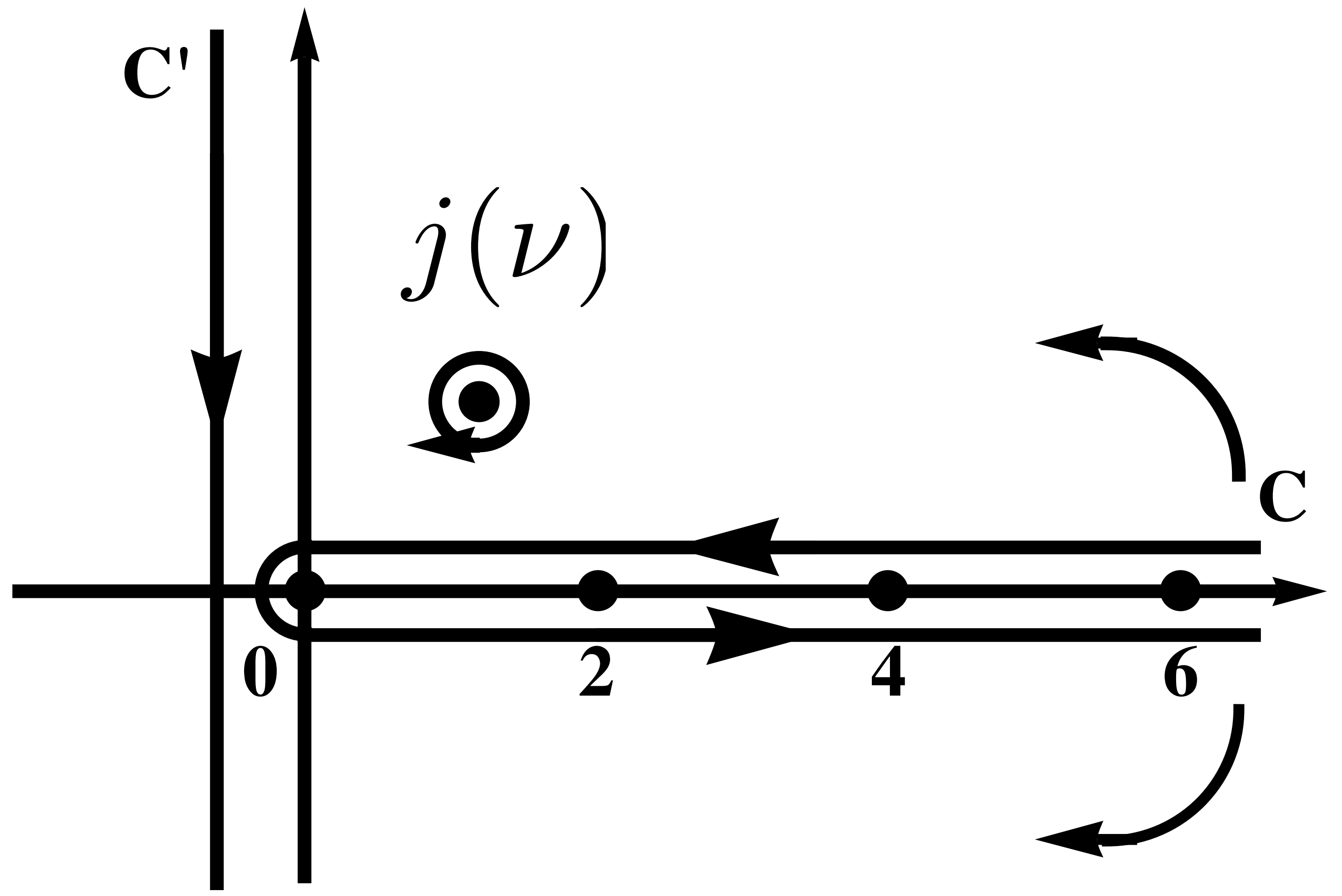

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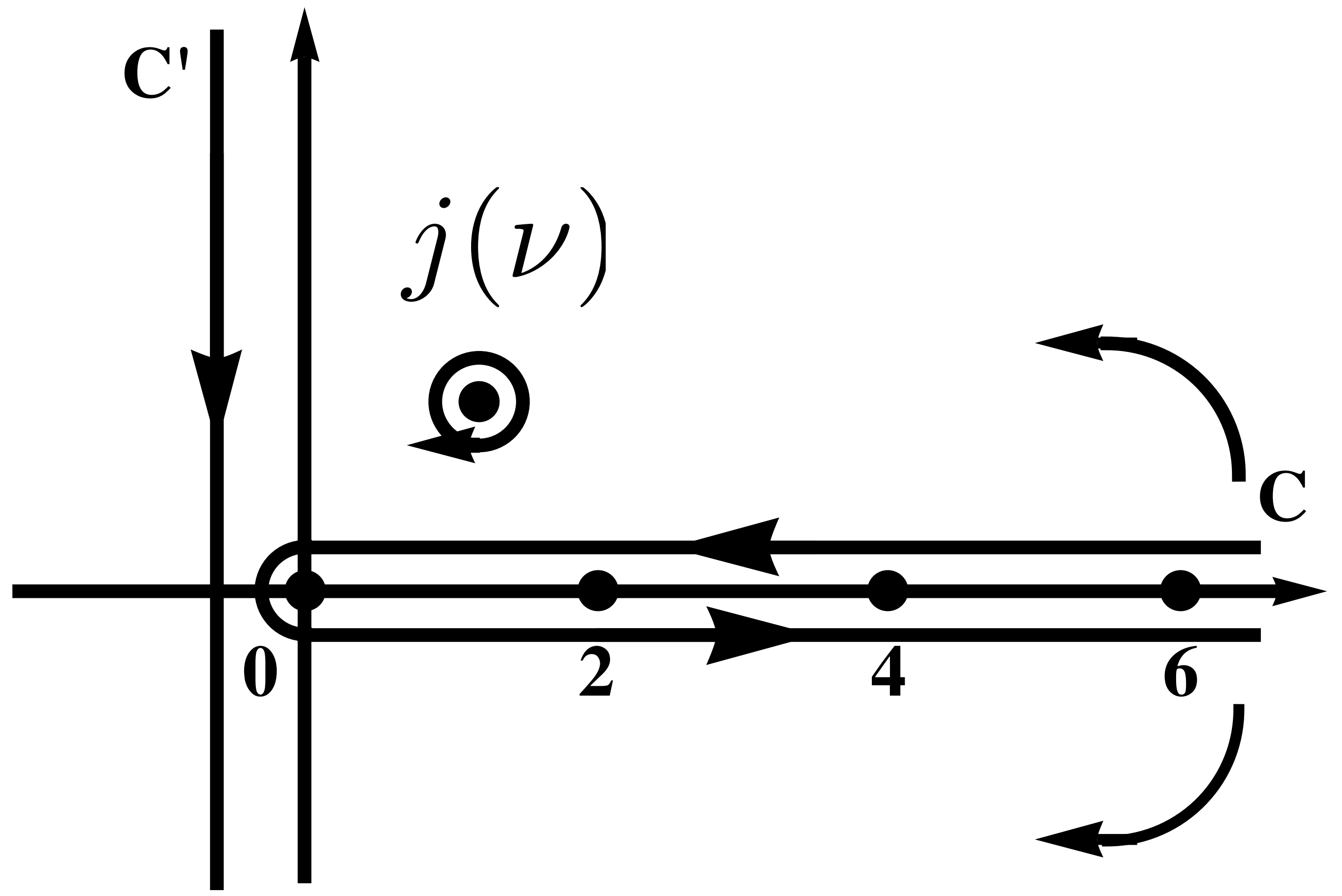

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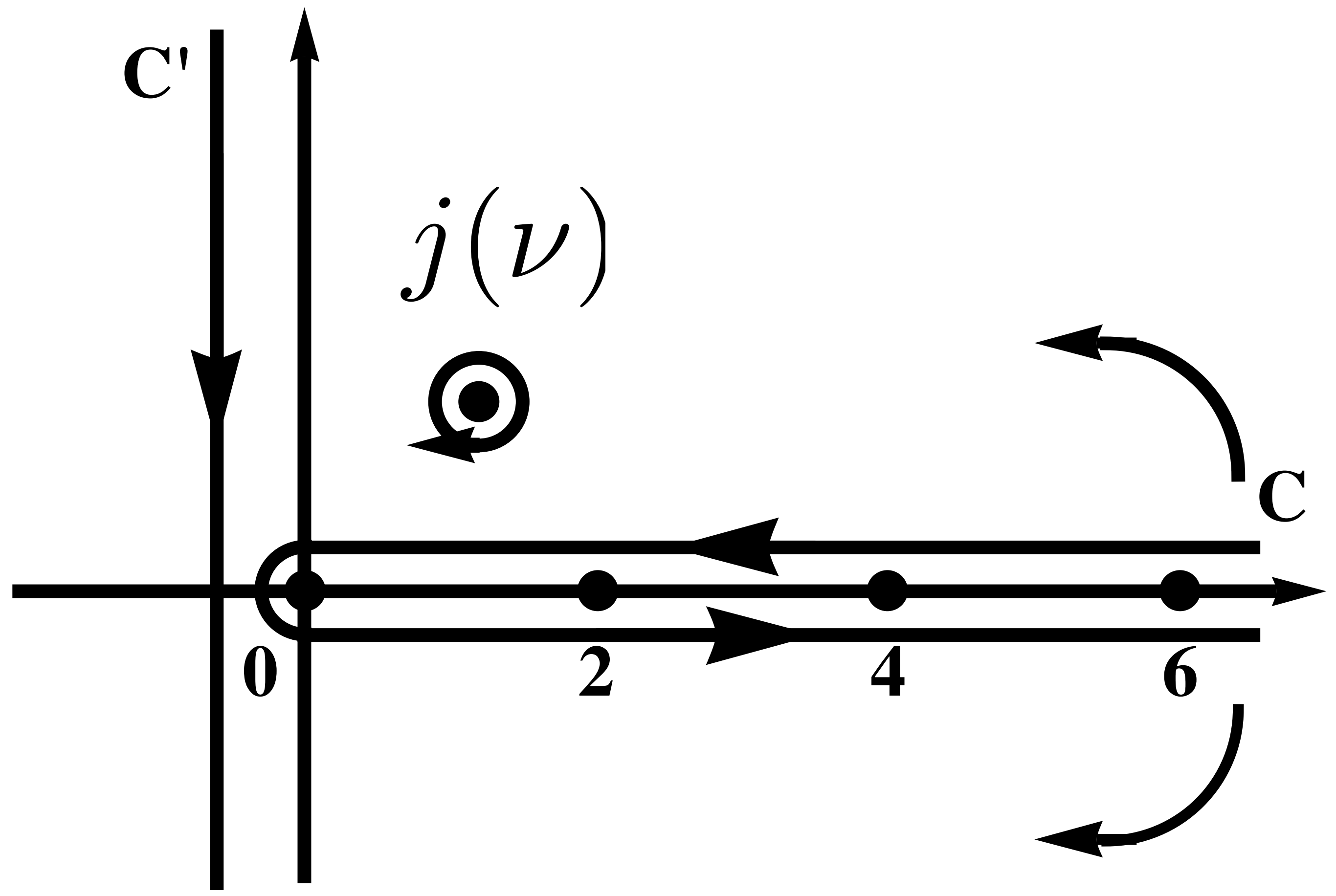

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$$\beta(\nu) \;\to\; C_{12j(\nu)} C_{34j(\nu)}$$

N=4 Super Yang Mills

- Correlation functions that exchange vacuum quantum numbers are dominated in Regge limit by exchange of pomeron/graviton Regge trajectory (twist 2)

$$\mathcal{O}_1 = \operatorname{tr}\left(\phi_{12}\phi^{12}\right)$$

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$$\mathcal{O}_L = \begin{cases} \operatorname{tr}\left(F_{\mu\nu_1}D_{\nu_2}\dots D_{\nu_{J-1}}F_{\nu_J}{}^{\mu}\right) \\ \operatorname{tr}\left(\phi_{AB}D_{\nu_1}\dots D_{\nu_J}\phi^{AB}\right) \\ \operatorname{tr}\left(\bar{\psi}_A D_{\nu_1}\dots D_{\nu_{J-1}}\Gamma_{\mu_J}\psi^{A}\right) \end{cases}$$

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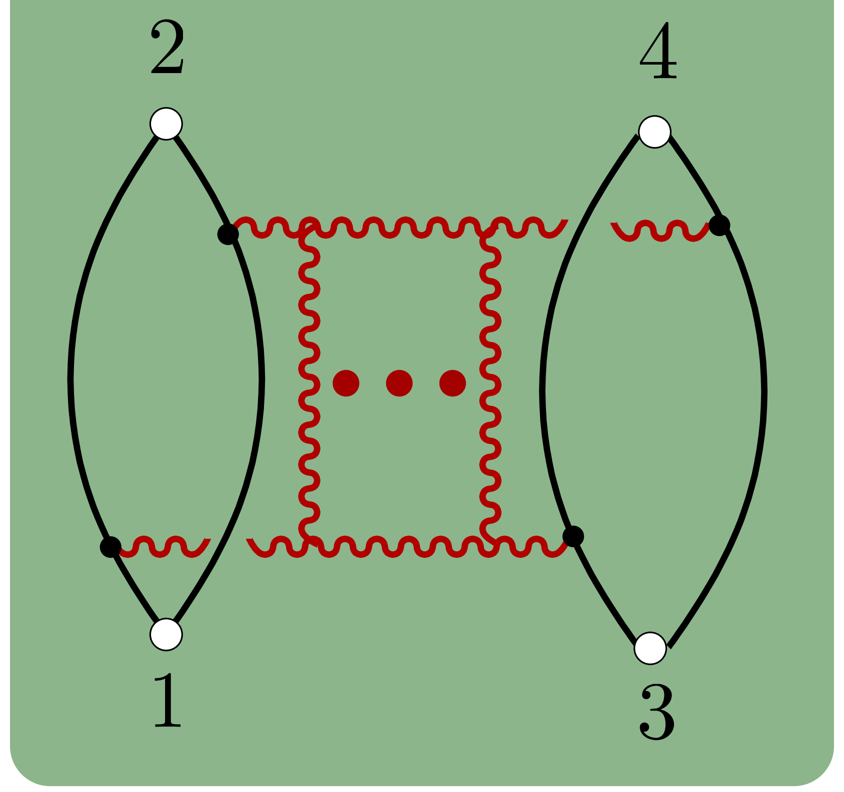
- Weak coupling

Pomeron

$g \ll 1$



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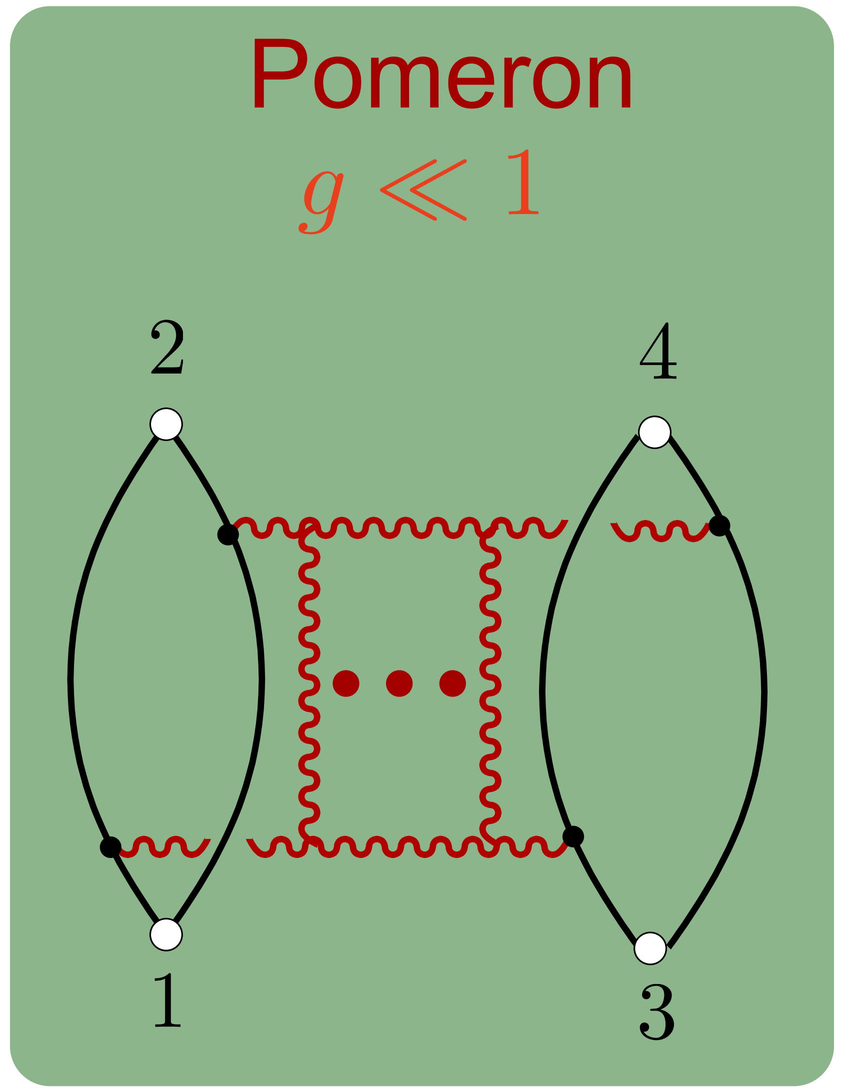
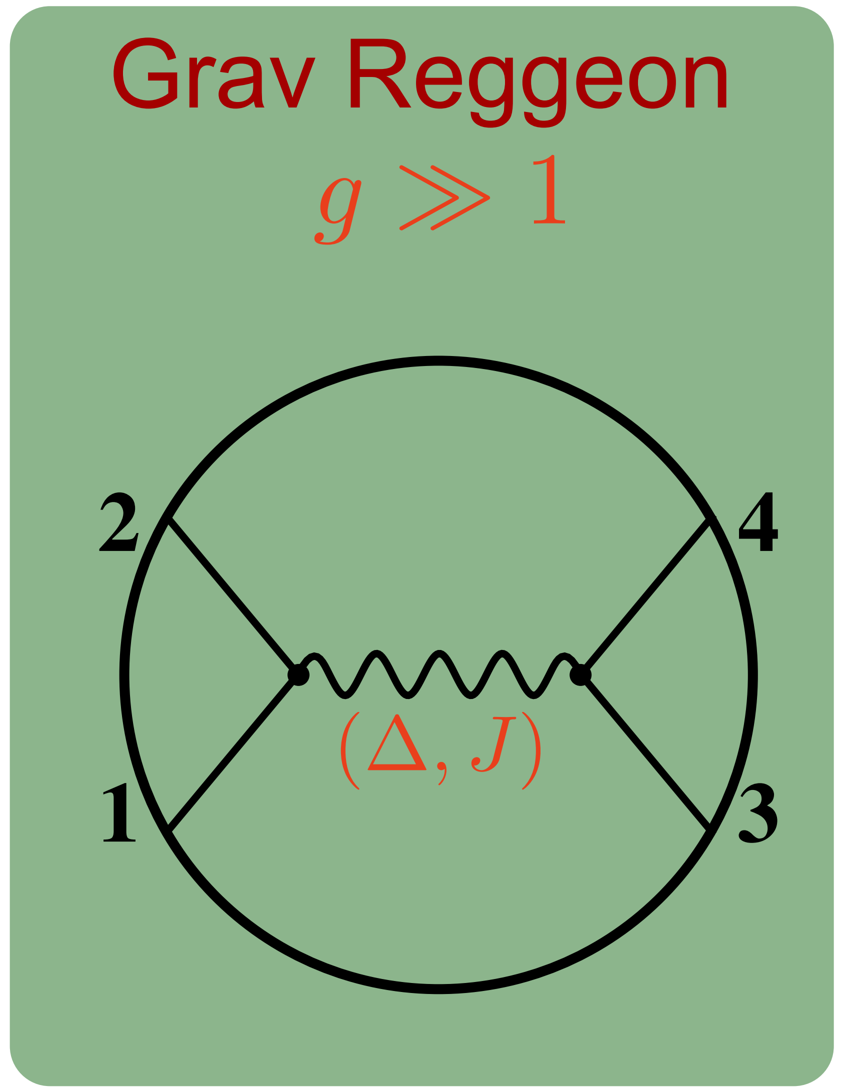
- Weak coupling
- Strong coupling





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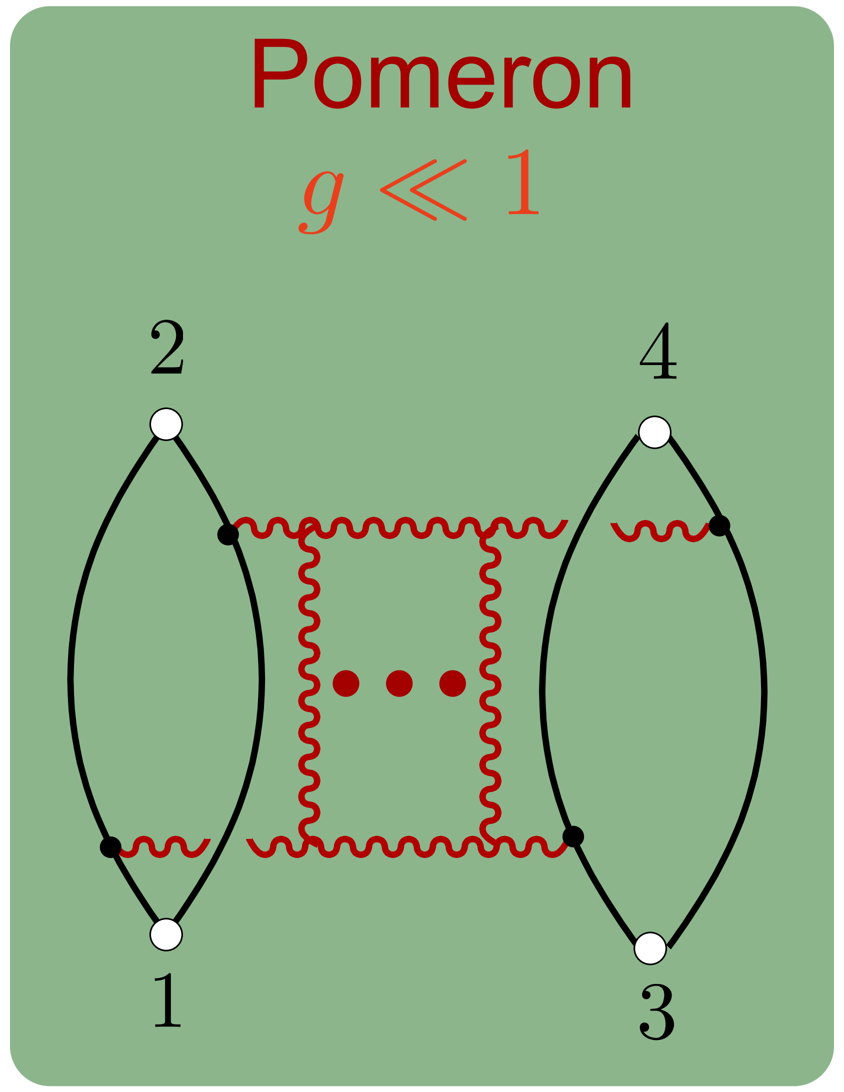
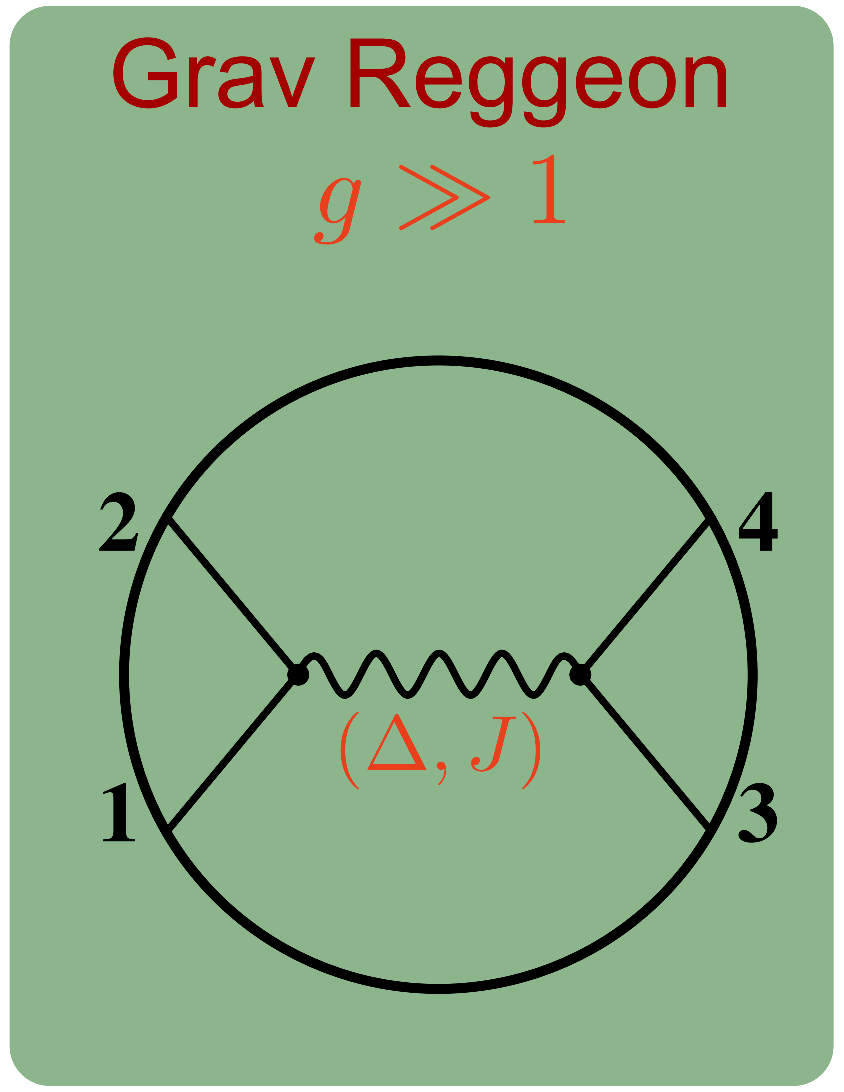




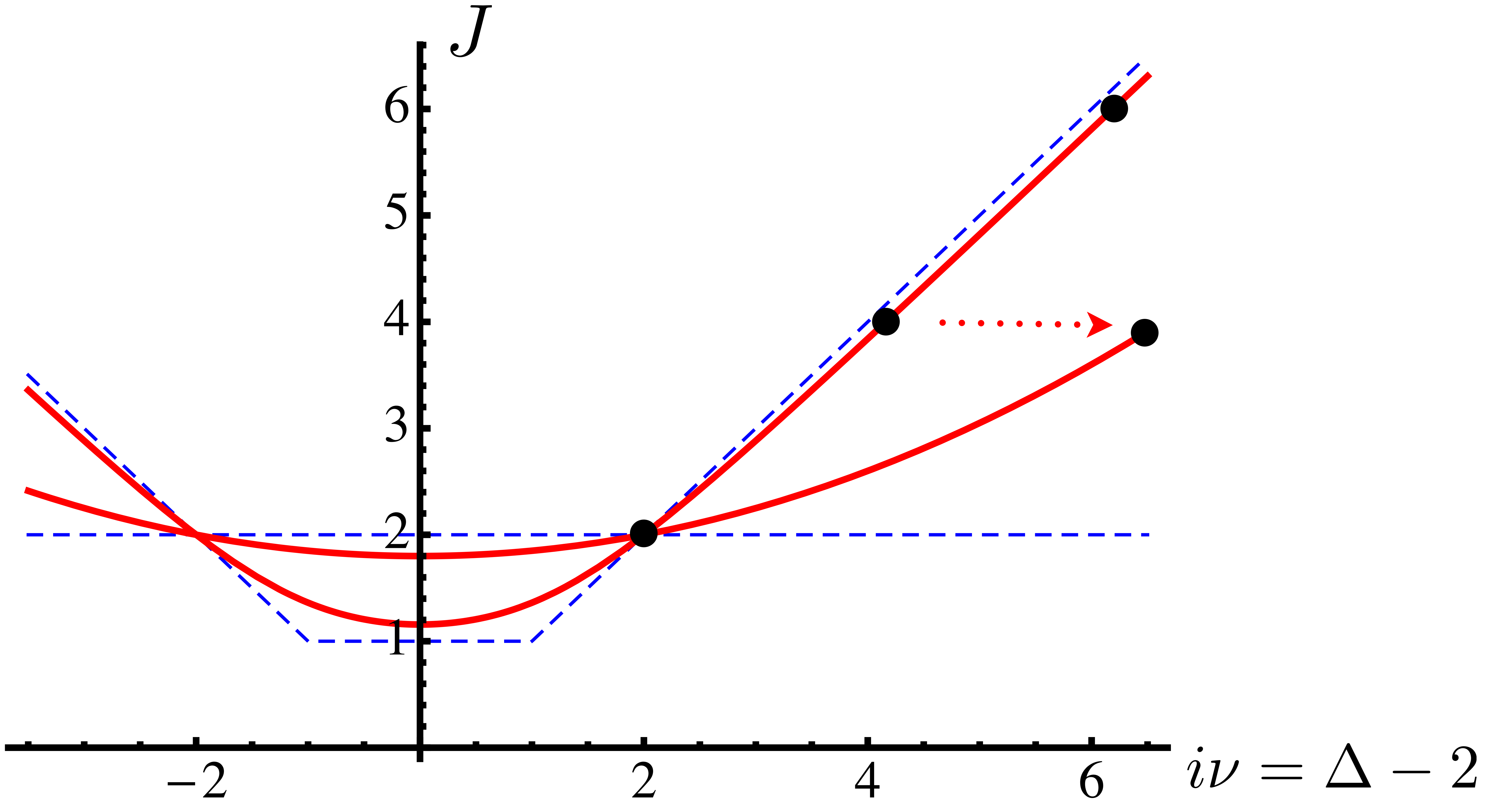
‘t Hooft coupling

$$\lambda = g_{YM}^2 N$$

$$= (4\pi g)^2$$

Reggeon spin & dimension of twist 2 operators

$\Delta = \Delta(J)$ or $J = j(\nu)$ $\qquad$ $\Delta\big(j(\nu)\big) = 2 + i\nu$



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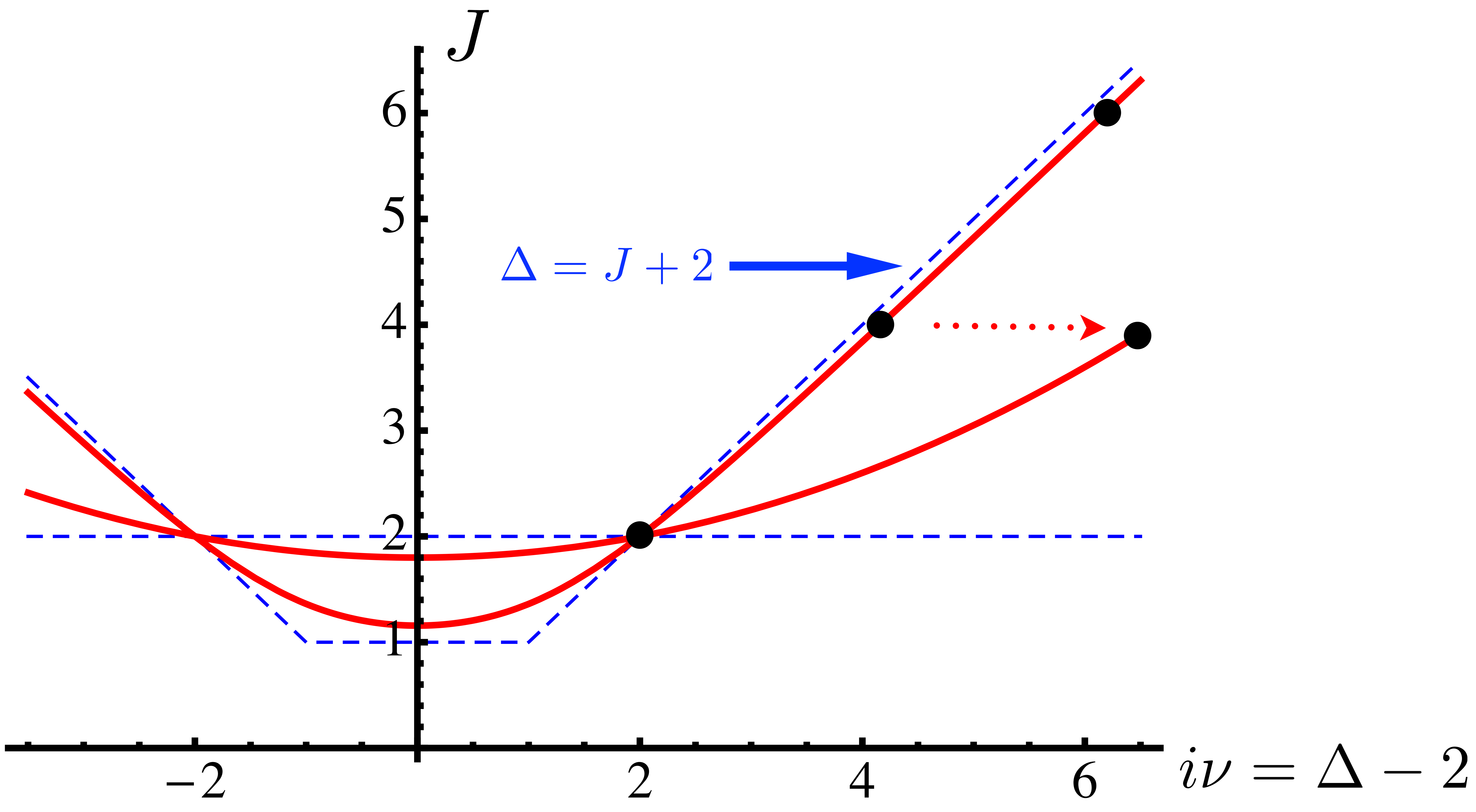

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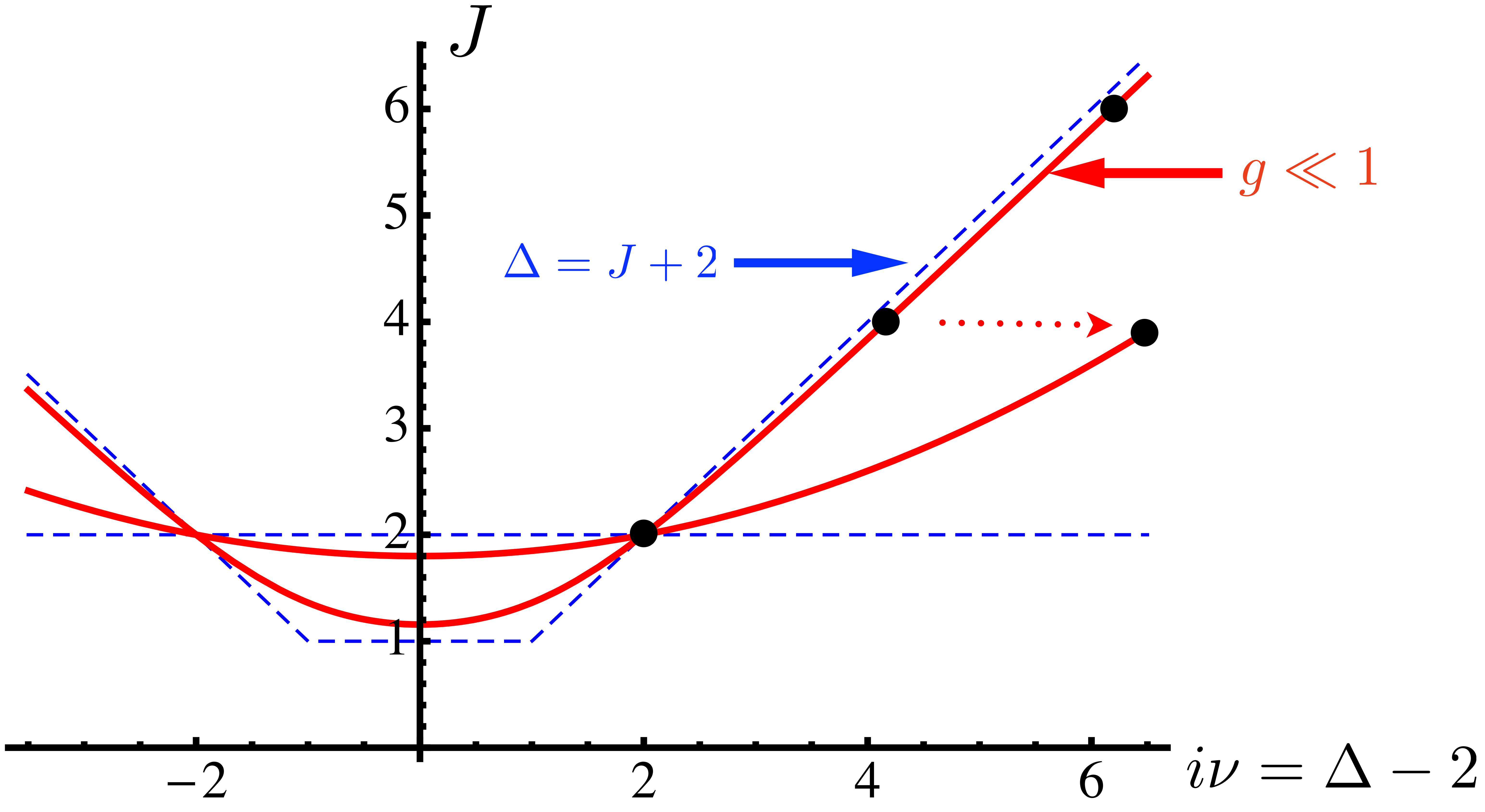

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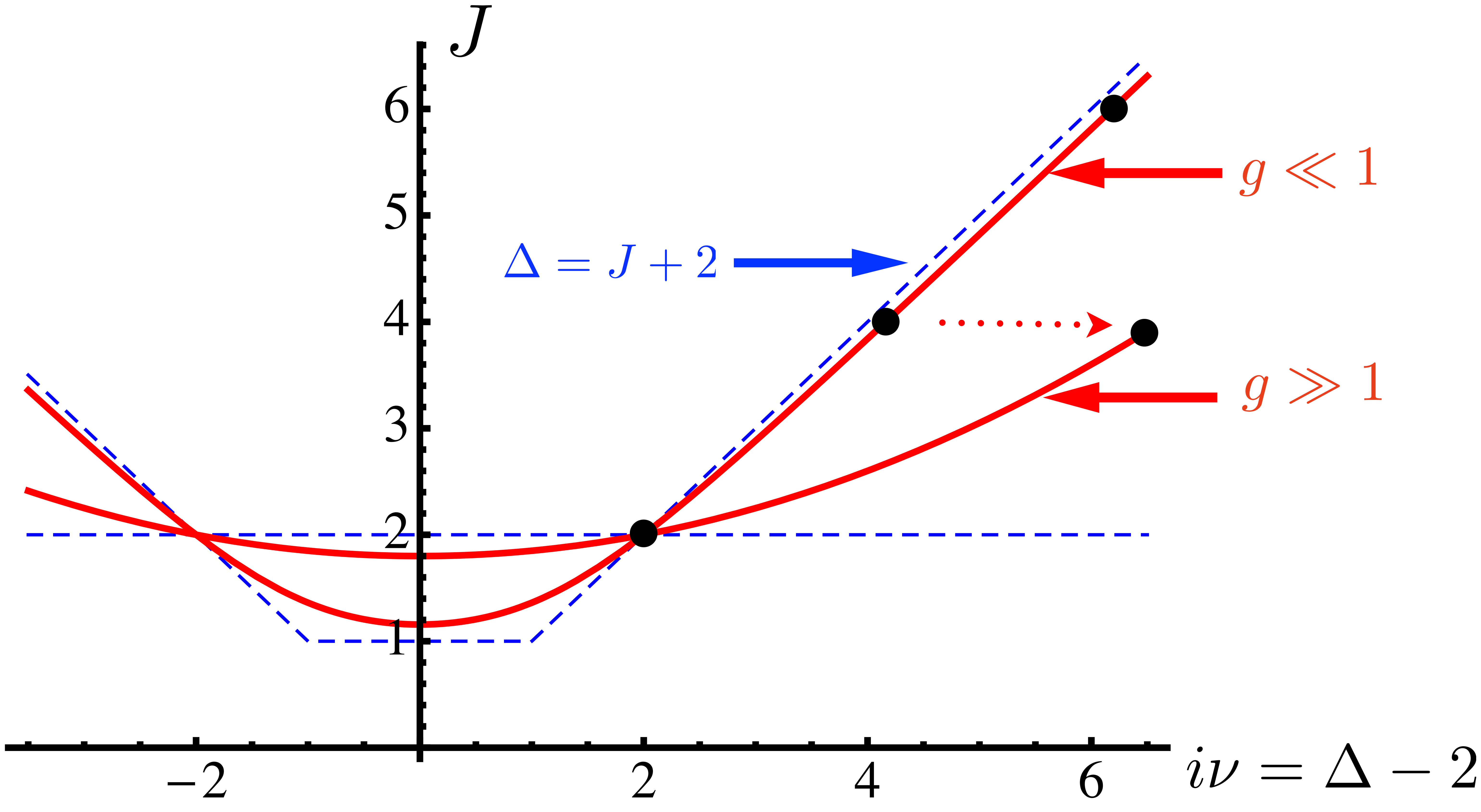

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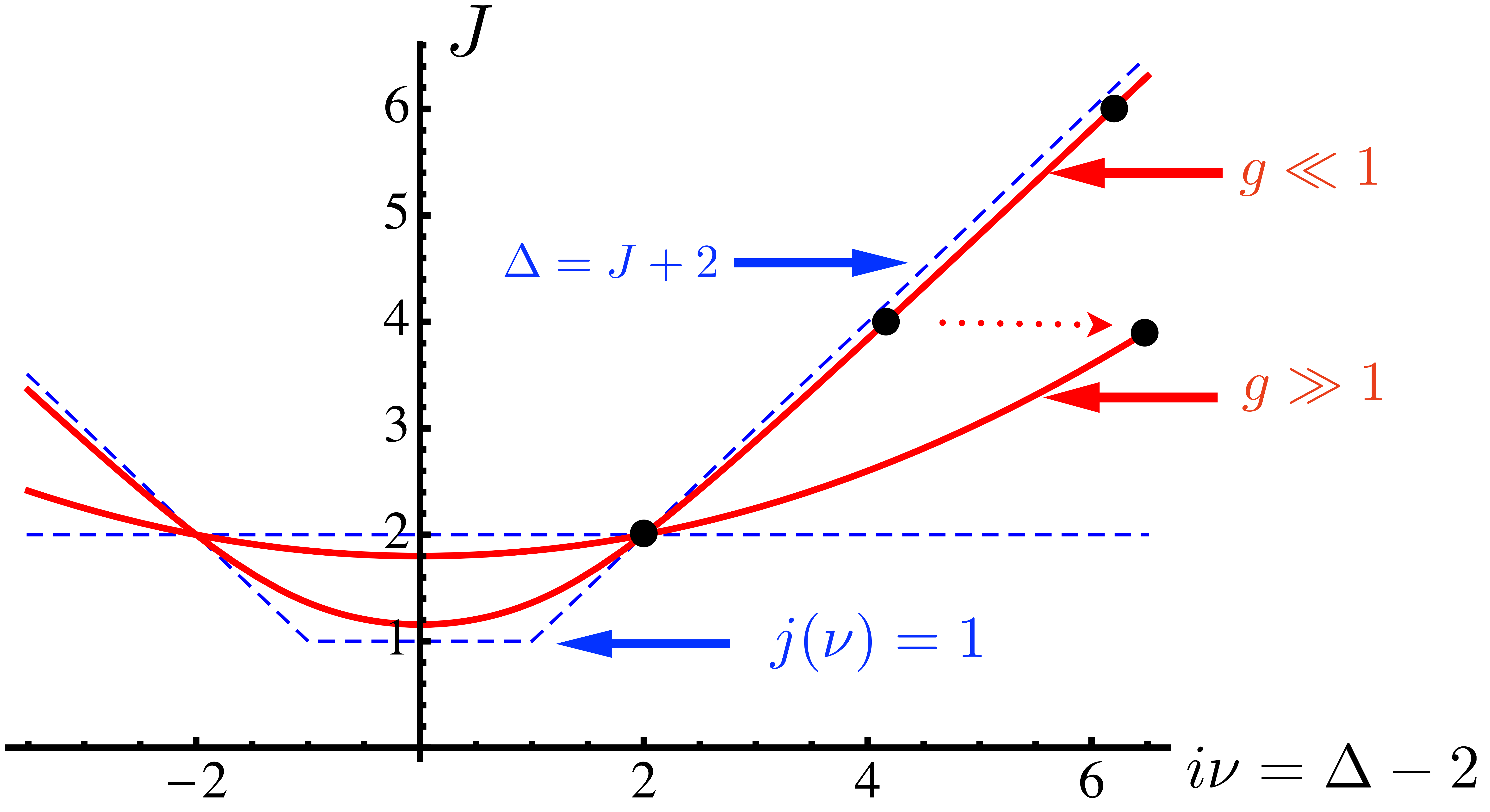

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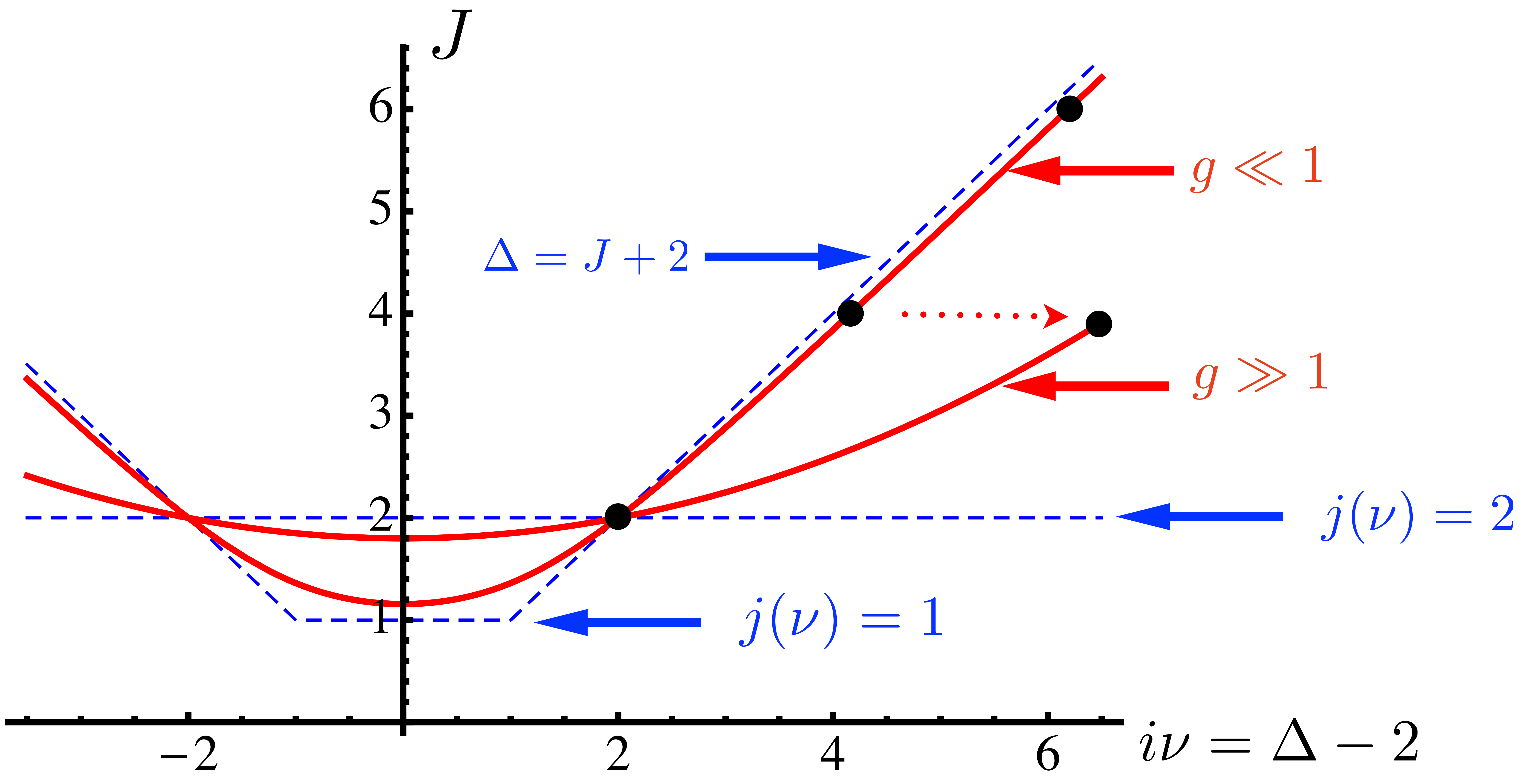

N=4 Super Yang Mills - anomalous dimension at weak coupling [Kotikov et al 07]

- Anomalous dimension (integrability) $\gamma(J) = \Delta(J) - J - 2 = \sum_{n=1}^{\infty} g^{2n}\gamma_n(J)$

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$$\Delta(J) - 3 = 2\left(\frac{-4g^2}{J-1}\right) + 0\left(\frac{-4g^2}{J-1}\right)^2 + 0\left(\frac{-4g^2}{J-1}\right)^3 - 4\zeta\,(3)\left(\frac{-4g^2}{J-1}\right)^4 + \cdots$$

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- NLO in BFKL spin gives next to leading behaviour near $J = 1$

N=4 Super Yang Mills - OPE coefficients at weak coupling

- From known form of 4pt correlation function at two loop obtain prediction for behaviour of OPE coefficients between external operators and operators in the leading Regge trajectory around $J = 1$ to arbitrary high order in coupling

$$\mathcal{O}_1 = \mathrm{tr}\left(\phi_{12}\phi^{12}\right)$$

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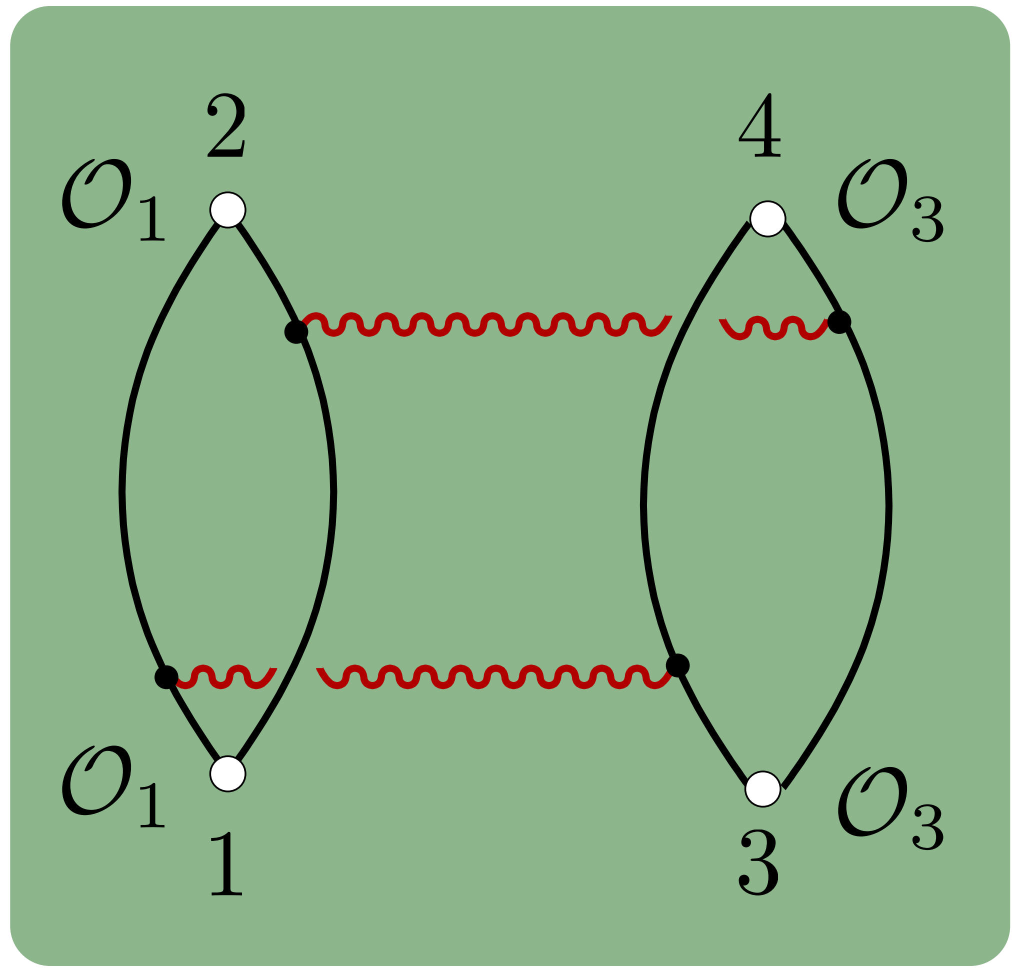

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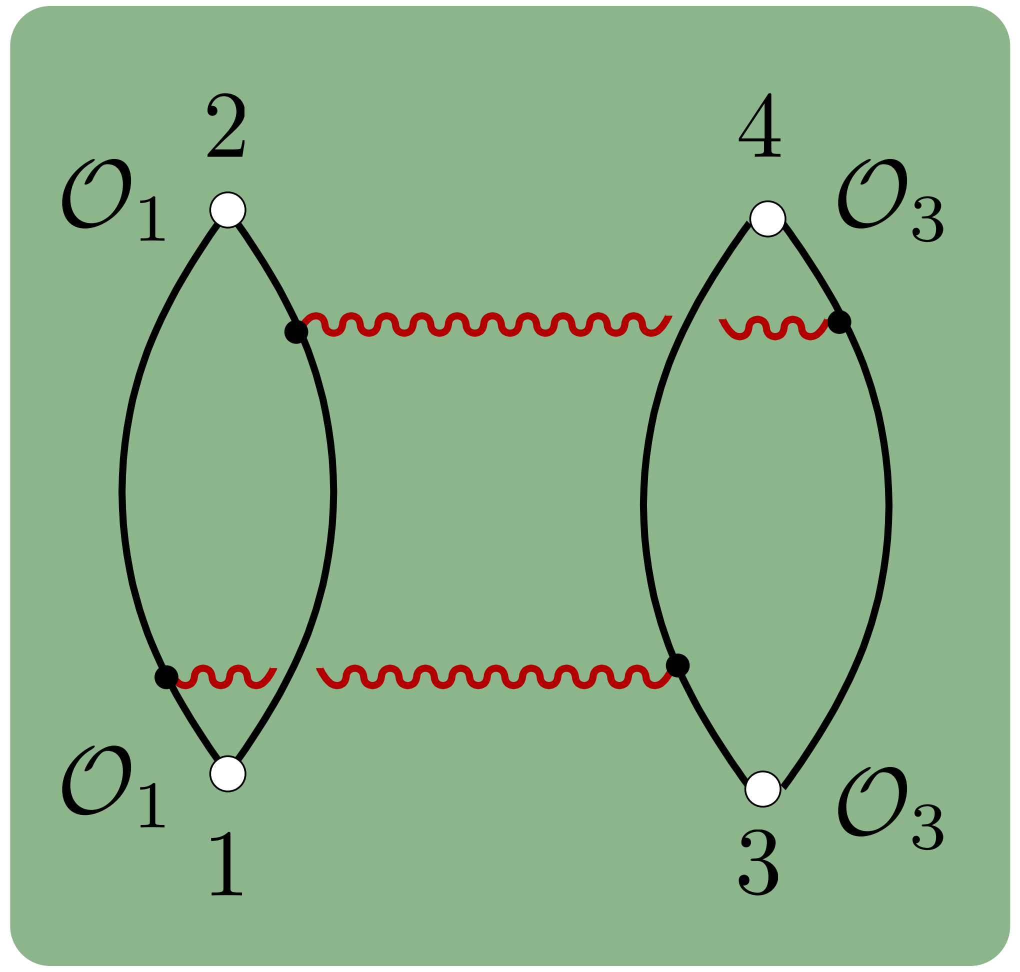
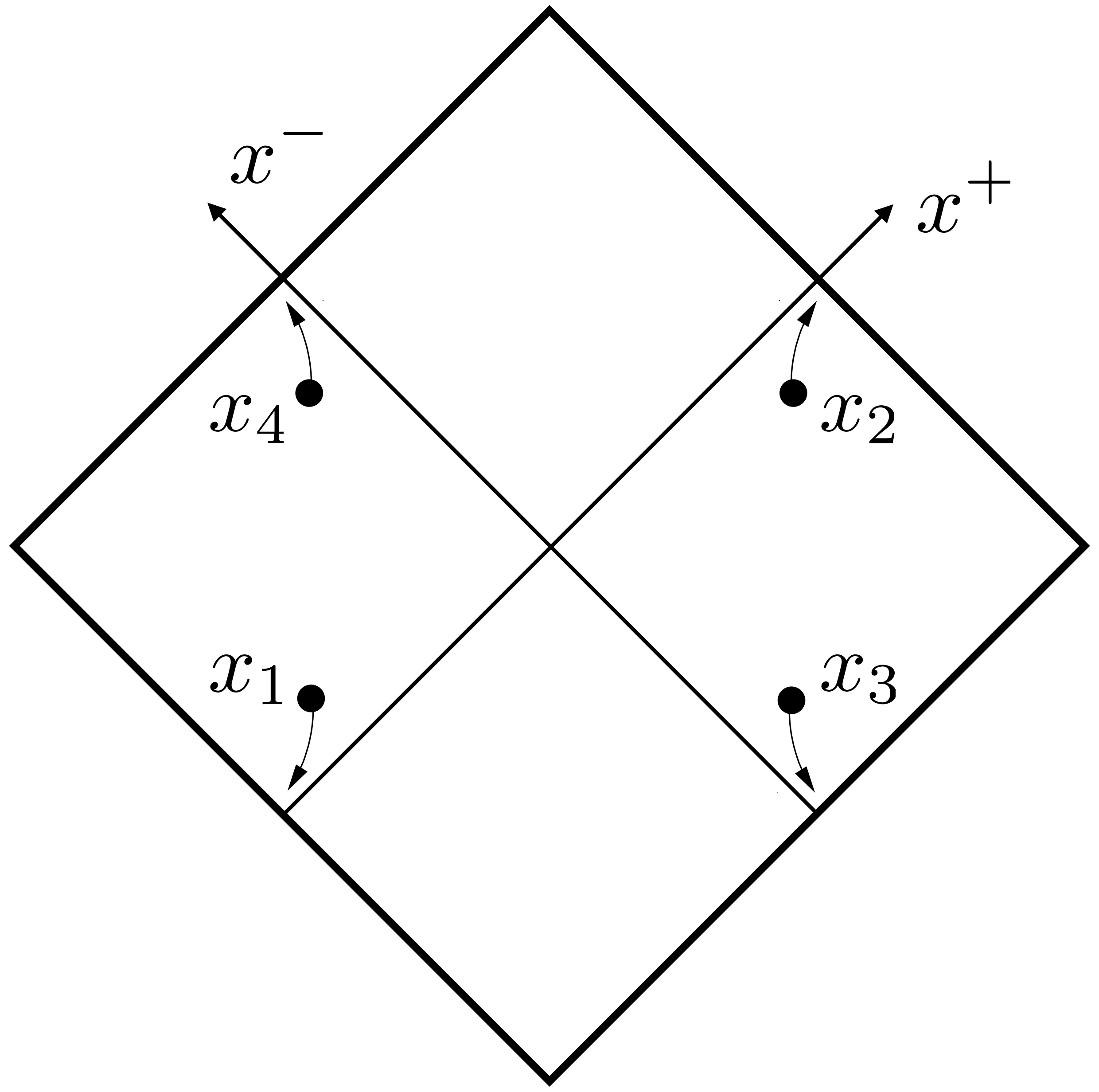


Regge limit in position space



$$\mathcal{A}(\sigma,\rho) \approx 2\pi i \int d\nu\, \alpha(\nu)\, \sigma^{1-j(\nu)}\Omega_{i\nu}(\rho)$$

$$\Big\downarrow$$

$$M(s,t) \approx \int d\nu\, \beta(\nu)\, \omega_{\nu,j(\nu)}(t)\, s^{j(\nu)}$$

$$\Big\downarrow$$

$$C_{11j(\nu)}C_{33j(\nu)}$$

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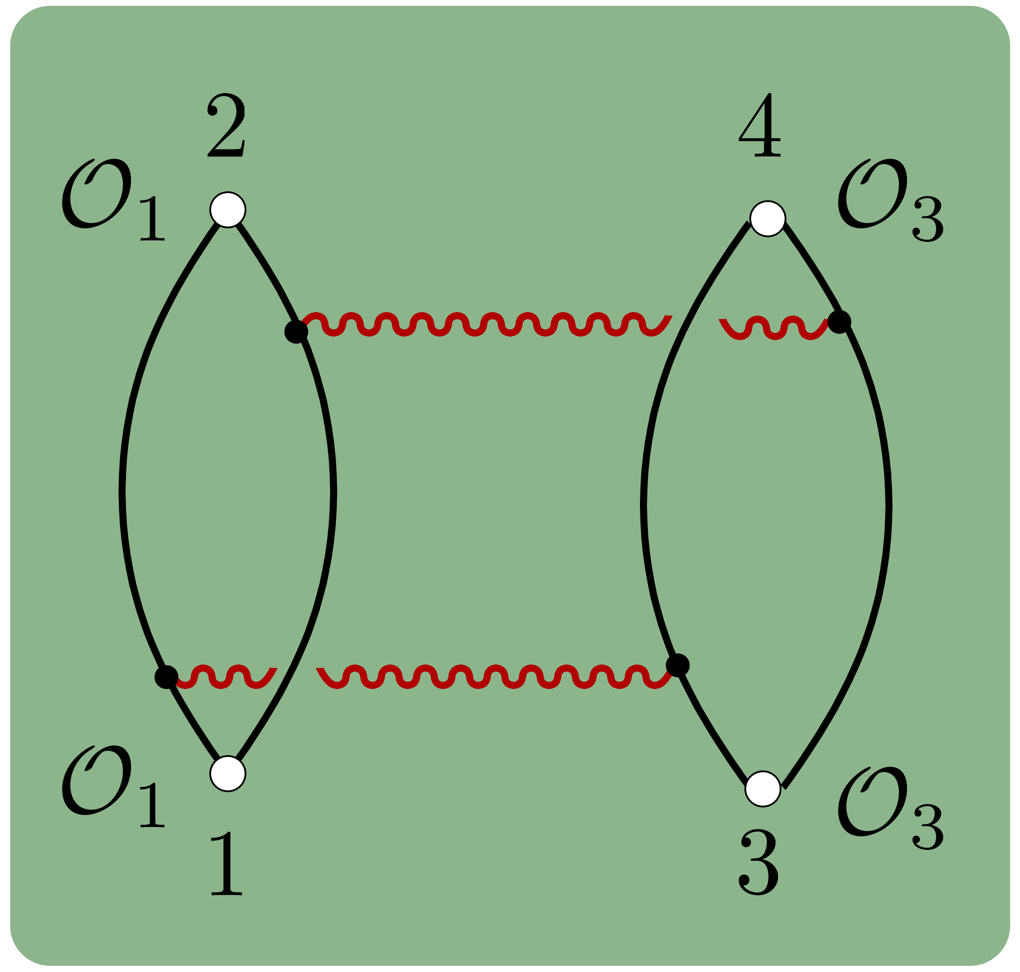
$$
\begin{aligned}
C_{11J}C_{33J} \quad &= \quad g^0\left[(J-1)\,\frac{2}{3}+O(J-1)^2\right]+\\
&\quad\ \ g^2\left[\frac{64}{9}+O(J-1)\right]+\\
&\quad\ \ g^4\left[\frac{1}{J-1}\,\frac{32}{27}\,\left(61-3\pi^2\right)+O(J-1)^0\right]+\\
&\qquad\qquad\qquad\qquad \cdots
\end{aligned}
$$

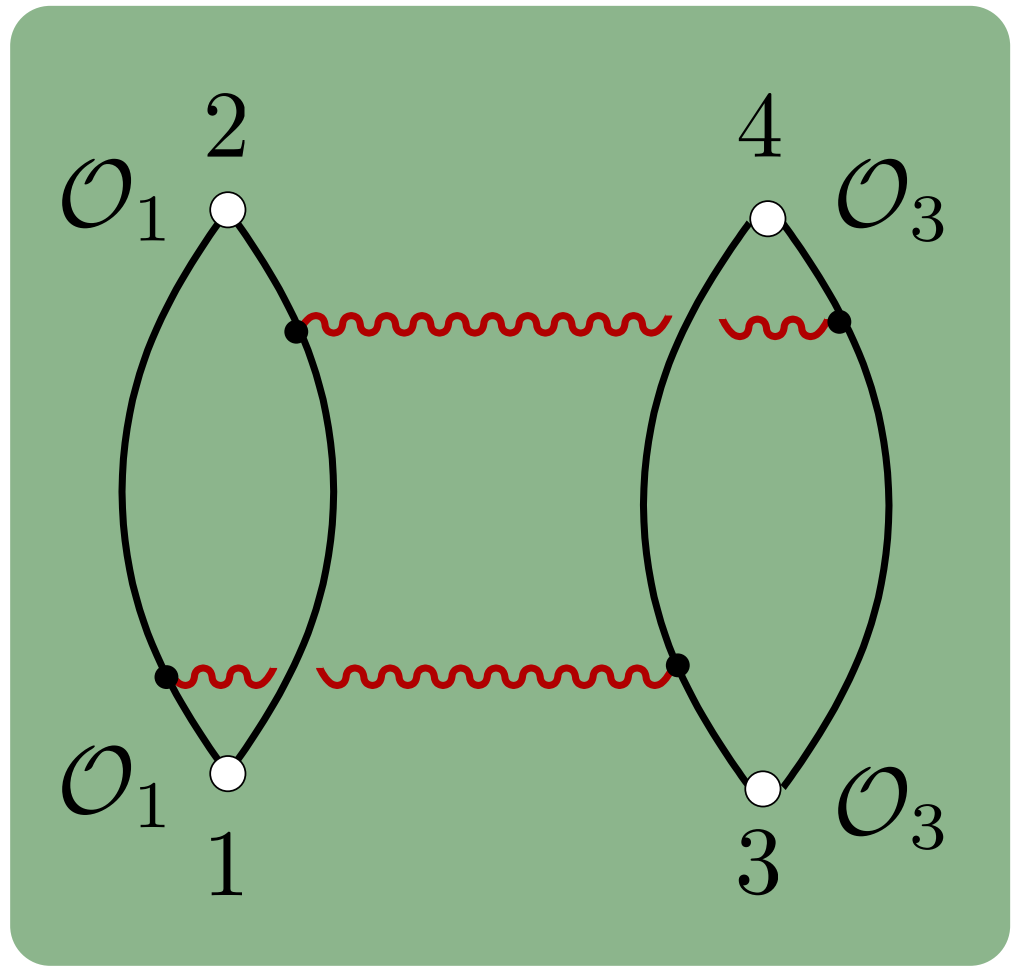


N=4 Super Yang Mills - OPE coefficients at weak coupling

- From known form of 4pt correlation function at two loop obtain prediction for behaviour of OPE coefficients between external operators and operators in the leading Regge trajectory around $J=1$ to arbitrary high order in coupling

$$\begin{aligned} C_{11J}C_{33J} \;=\;& g^0\left[\boxed{(J-1)\,\frac{2}{3}}+O(J-1)^2\right]+ \\ & g^2\left[\frac{64}{9}+O(J-1)\right]+ \\ & g^4\left[\frac{1}{J-1}\,\frac{32}{27}\,\left(61-3\pi^2\right)+O(J-1)^0\right]+ \\ & \qquad\qquad \dots \end{aligned}$$

Free theory (Wick contractions)



- Next to leading order correlation function also known [Balitsky, Chirilli 08]

$$
\begin{aligned}
C_{11J}C_{33J} \quad = \quad & g^0\left[(J-1)\,\frac{2}{3}+(J-1)^2\,\frac{2}{9}\,\big(-8+3\ln(2)\big)+O(J-1)^3\right]+ \\
& g^2\left[\frac{64}{9}+(J-1)\,\frac{2}{27}\,\big(-488+9\pi^2+96\ln 2\big)+O(J-1)^2\right]+ \\
& g^4\left[\frac{1}{J-1}\,\frac{32}{27}\,\left(61-3\pi^2\right)+\quad\cdots\quad+O(J-1)\right]+ \\
& \qquad\qquad\qquad \ldots
\end{aligned}
$$

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Free theory (Wick contractions)

$$
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C_{11J}C_{33J} \quad = \quad & g^0 \left[\boxed{(J-1)\,\frac{2}{3} + (J-1)^2\,\frac{2}{9}\Big(-8 + 3\ln(2)\Big)} + O(J-1)^3 \right] + \\
& g^2 \left[\frac{64}{9} + (J-1)\,\frac{2}{27}\Big(-488 + 9\pi^2 + 96\ln 2\Big) + O(J-1)^2 \right] + \\
& g^4 \left[\frac{1}{J-1}\,\frac{32}{27}\,\big(61 - 3\pi^2\big) + \quad \cdots \quad + O(J-1) \right] +
\end{aligned}
$$

$$\ldots$$

N=4 Super Yang Mills - Reggeon spin at strong coupling

- Anomalous dimension of string states in leading Regge trajectory know up to next to next leading order [Gromov et al 11]

$x = J - 2$

$$\Delta(J)(\Delta(J)-4) = x\left[2\sqrt{\lambda} + \left(-1+\frac{3x}{2}\right) - \frac{3}{8}\Big(-10 + x(8\zeta(3)-1) + x^2\Big)\frac{1}{\sqrt{\lambda}} + \cdots\right]$$

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- Can invert, $\Delta(j(\nu)) = 2 + i\nu$, to learn about behaviour of graviton Regge trajectory around $J=2$ to arbitrary high order in strong coupling expansion

$$j(\nu) = 2 - \frac{4+\nu^2}{2\sqrt{\lambda}}\left(1 + \sum_{n=2}^{\infty}\frac{\tilde{j}_n(\nu^2)}{\lambda^{(n-1)/2}}\right)$$

$\tilde{j}_n(\nu^2)$ is a polynomial of degree $n-2$ [Cornalba 07]

$$\tilde{j}_n(\nu^2) = \sum_{k=0}^{n-2} c_{n,k}\nu^{2k}$$

In flat space limit

$\nu^2 = -R^2 T \sim \lambda^{1/2}$

$$c_{2,0} = \frac{1}{2}\,, \quad c_{3,0} = -\frac{1}{8}\,, \quad c_{3,1} = \frac{3}{8}\,, \quad c_{4,1} = -\frac{3}{32}\big(8\zeta(3)-7\big)\,, \quad c_{5,2} = \frac{21}{64}\,, \quad c_{n,k} = 0 \text{ for } \left[\frac{n}{2}\right] \le k \le n-2 \text{ with } n \ge 4$$

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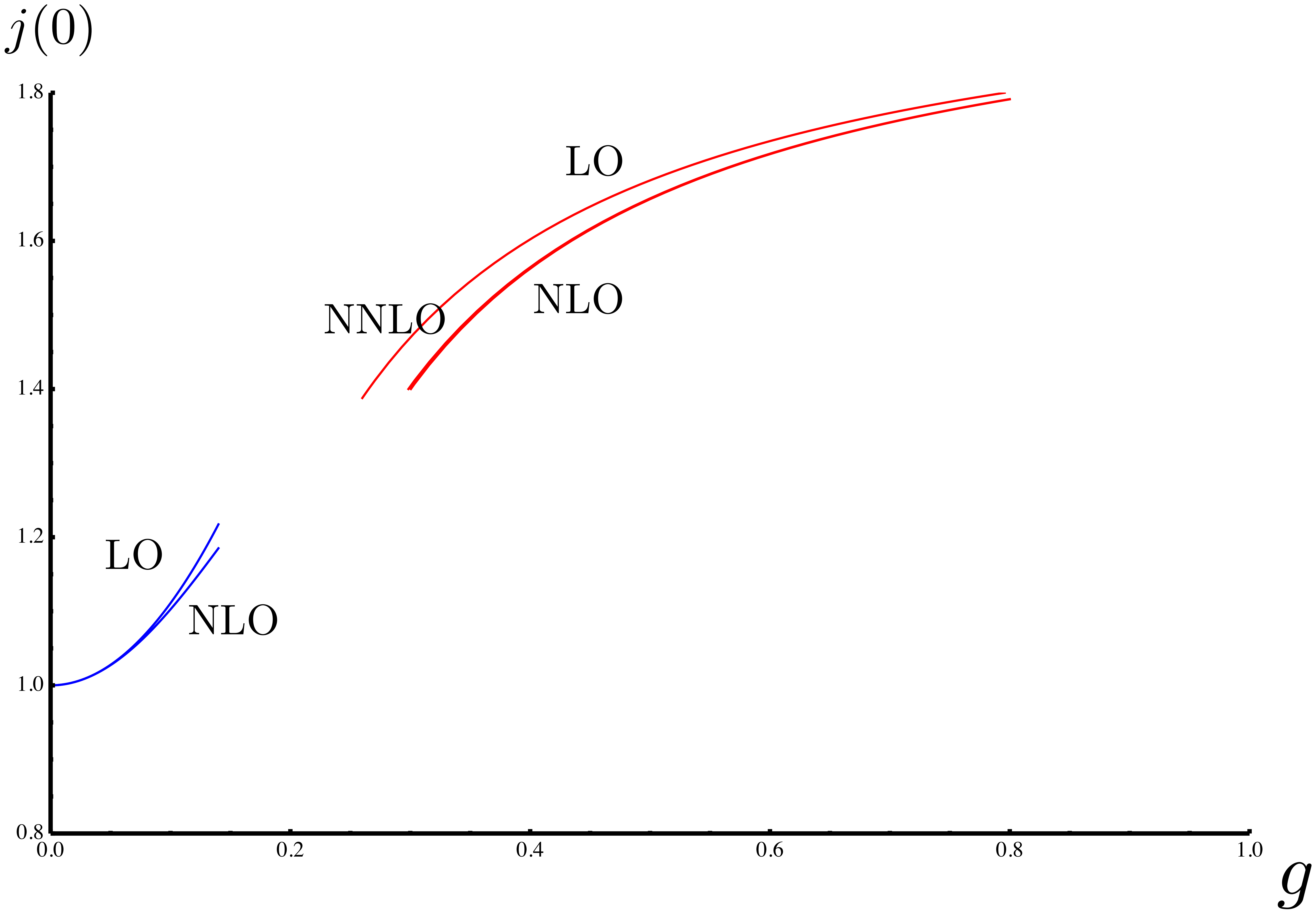
[Janik, work in progress]

N=4 Super Yang Mills - Reggeon spin at strong coupling

- New prediction for the strong coupling expansion of intercept

$$j(0) = 2 - \frac{2}{\sqrt{\lambda}}\left(1 + \boxed{\frac{1}{2\sqrt{\lambda}} - \frac{1}{8\lambda}}\right)$$

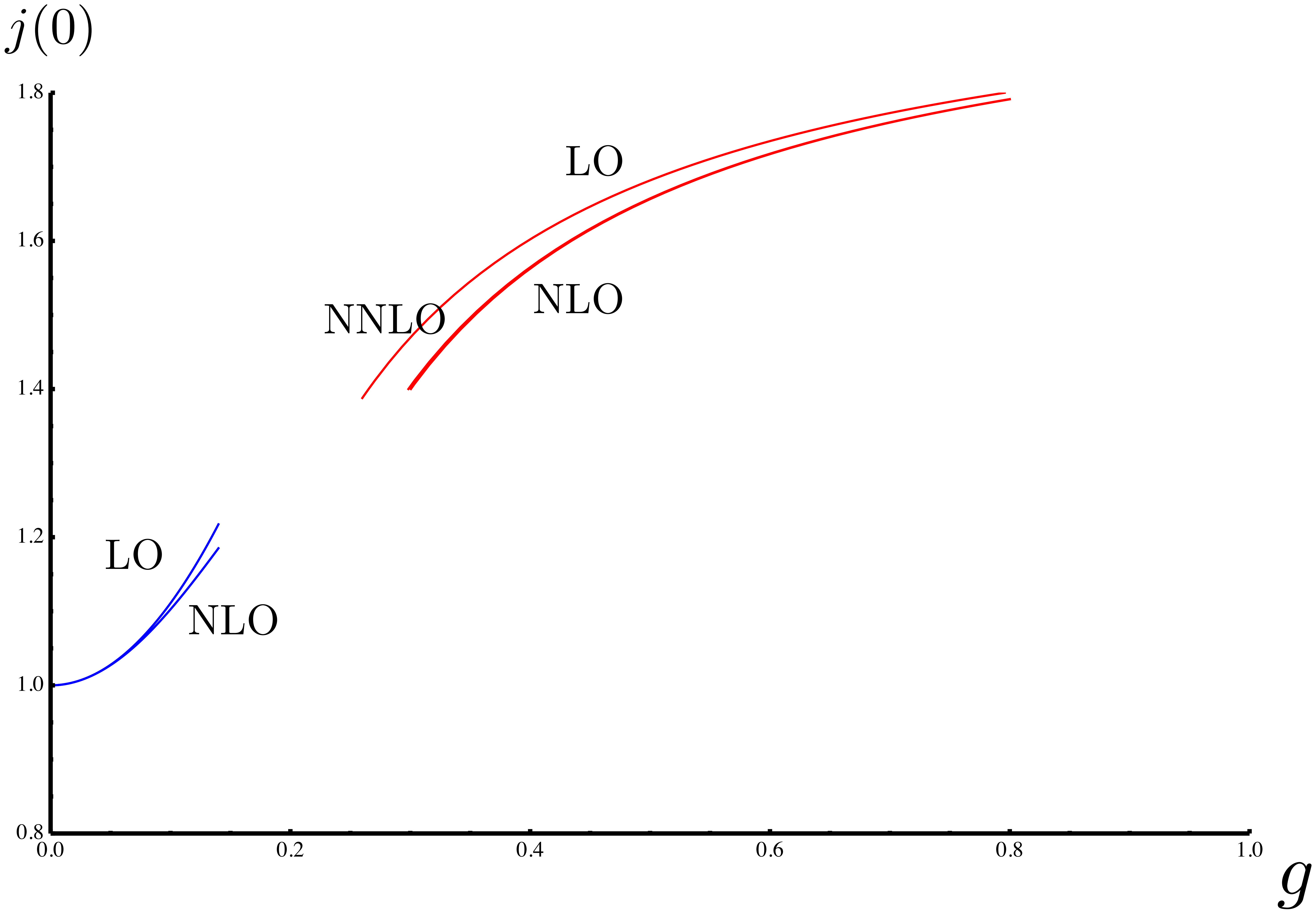


N=4 Super Yang Mills - Reggeon spin at strong coupling

- New prediction for the strong coupling expansion of intercept

$$j(0) = 2 - \frac{2}{\sqrt{\lambda}}\left(1 + \frac{1}{2\sqrt{\lambda}} - \frac{1}{8\lambda}\right) + 2(1-\zeta_3)\frac{1}{\lambda^2}$$

[Kotikov and Lipatov 13]



N=4 Super Yang Mills - OPE coefficients at strong coupling

- Flat space limit of Witten diagram [Penedones 10]

$$T(S,T) = \frac{1}{\mathcal{N}} \lim_{R\to\infty} V(S^5)\, R \int_{-i\infty}^{i\infty} \frac{d\alpha}{2\pi i}\, \alpha^{2-\frac{\sum_i \Delta_i}{2}}\, e^{\alpha}\, \underbrace{M\left(s = \frac{R^2 S}{2\alpha}, t = \frac{R^2 T}{2\alpha}\right)}_{M(s,t) \approx \int d\nu\, \beta(\nu)\, \omega_{\nu,j(\nu)}(t)\, s^{j(\nu)}}$$

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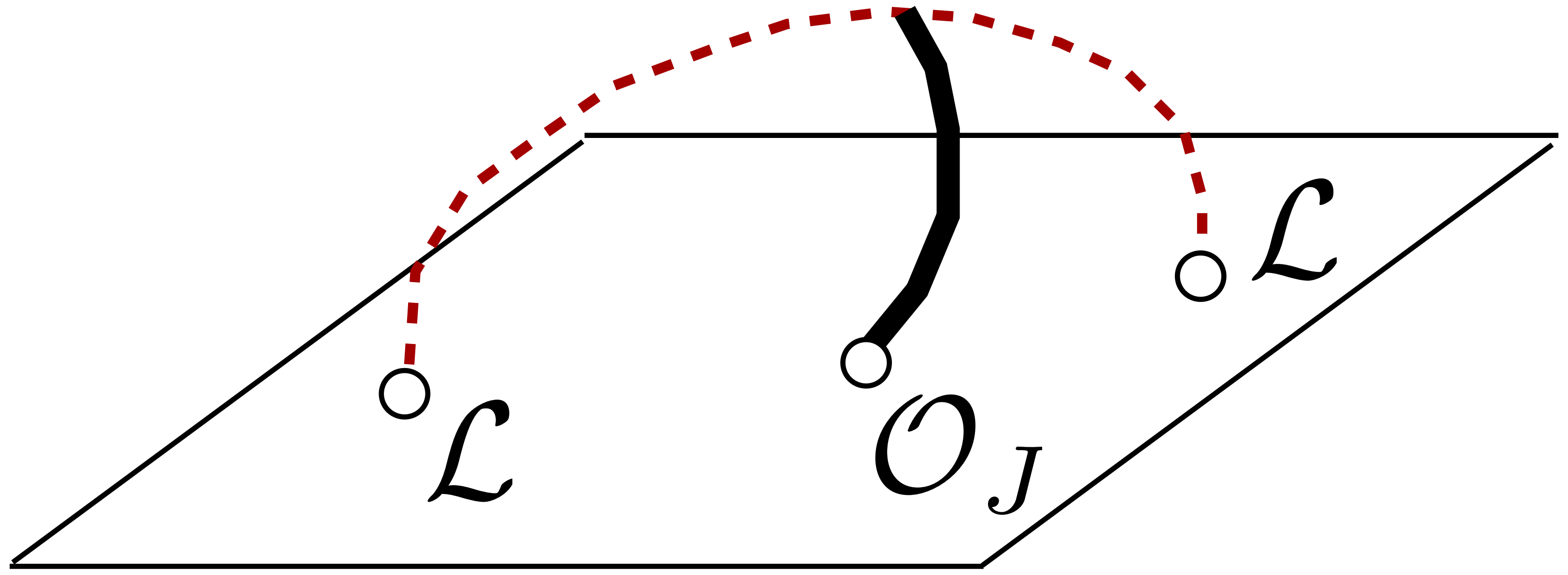
- To obtain Virasoro-Shapiro with linear trajectory restricts degree of $\tilde{j}_n(\nu)$

$$j(\nu) = 2 - \frac{4+\nu^2}{2\sqrt{\lambda}}\left(1 + \sum_{n=2}^{\infty} \frac{\tilde{j}_n(\nu^2)}{\lambda^{(n-1)/2}}\right) \quad \longrightarrow \quad J(T) = 2 + \frac{\alpha'}{2} T$$

N=4 Super Yang Mills - OPE coefficients at strong coupling

- Equating to Virasoro-Shapiro in Regge limit make prediction for strong coupling OPE coefficients involving Lagrangian and operators in leading Regge trajectory

$$C_{\mathcal{L}\mathcal{L}J} = \frac{\pi^{\frac{3}{2}}}{3N} \frac{(J-2)^{\frac{5+J}{2}}}{2^{1+J}\Gamma\left(\frac{J}{2}\right)} \lambda^{\frac{7}{4}} \, 2^{-\lambda^{1/4}\sqrt{2(J-2)}}$$



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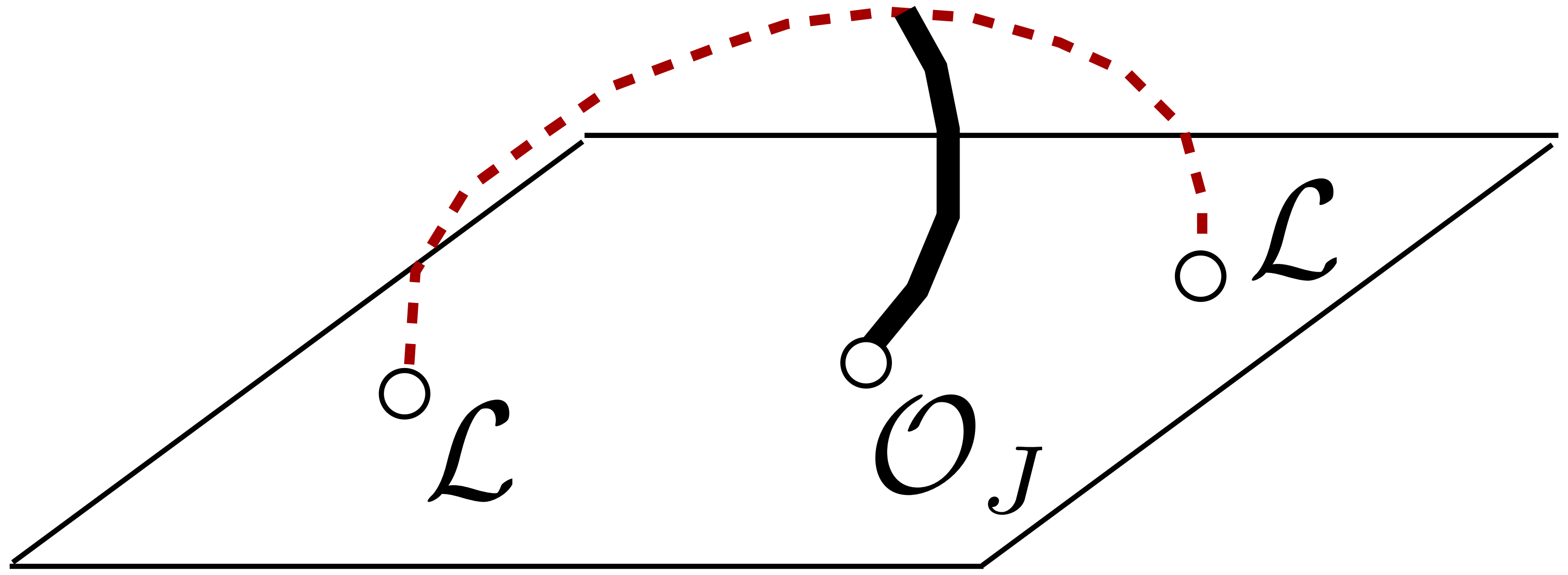

[Minahan 12]

Concluding remarks & future directions

- Extended Mellin technology for CFT’s to define Regge theory rigorously and in full analogy with flat space

- Explored consequences of Conformal Regge theory in N=4 SYM and gave many new predictions - useful data for program of solving theory exactly using integrability

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  - Study other trajectories. For example $\big\langle (XZ)(x_1)\,\big(\bar{X}Z\big)(x_2)\,\big(Y\bar{Z}\big)(x_3)\,\big(\bar{Y}\bar{Z}\big)(x_4)\big\rangle$

  would give information about OPE coefficient $\big\langle \big(Y\bar{Z}\big)(x_1)\,\big(\bar{Y}\bar{Z}\big)(x_2)\,\big(ZD^JZ\big)(x_3)\big\rangle$

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- One can interpolate between a CFT in UV and a confined gauge theory in IR where standard Regge theory applies. Can we understand better how conformal and standard Regge theories interpolate? (single Regge trajectory becomes infinite sequence of trajectories)

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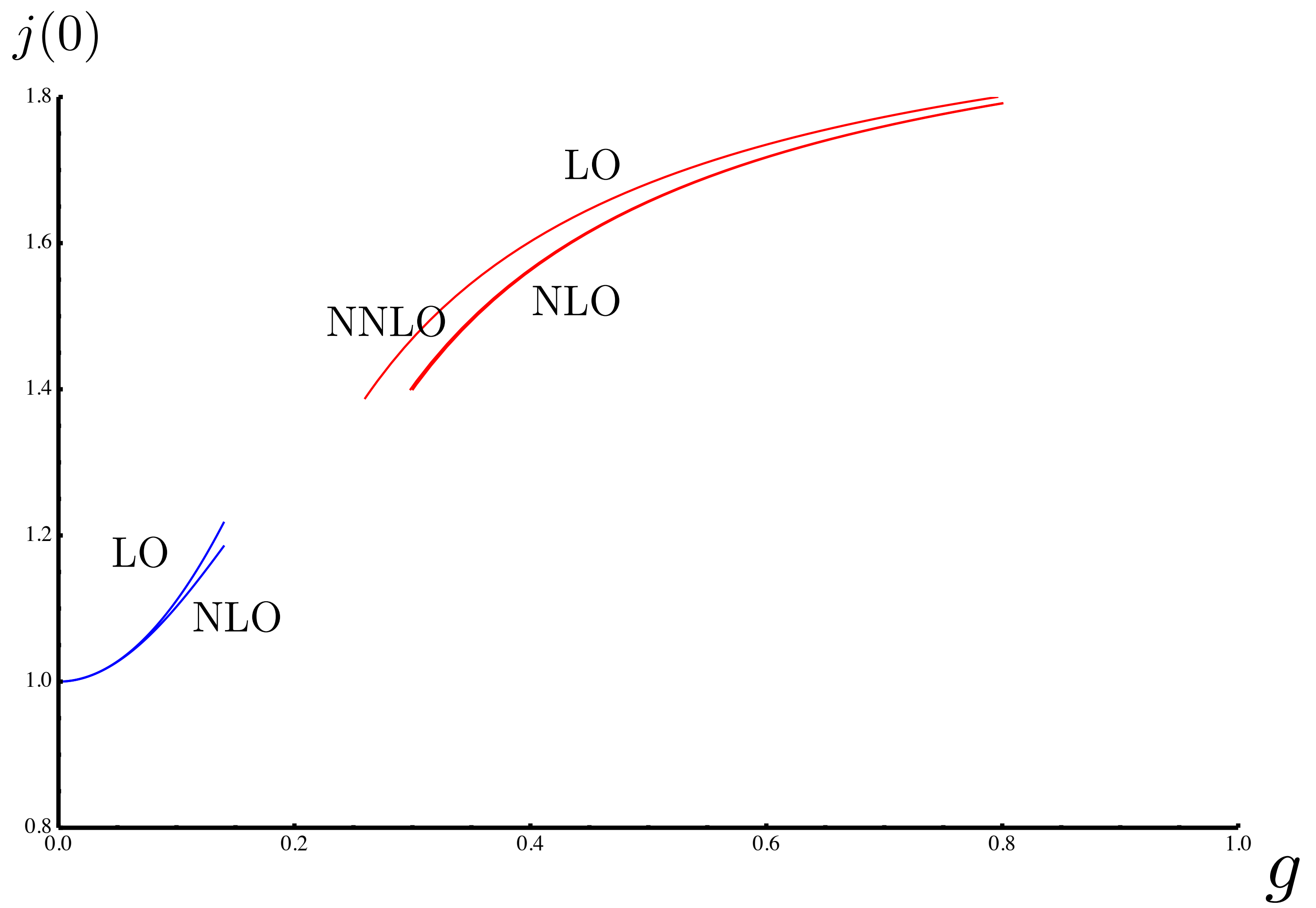
- In N=4 SYM can we derive spin of pomeron/graviton Regge trajectory using integrability for any value of the coupling (like Y-system for anomalous dimensions)?

[Janik]



THANK YOU