Thermal brane probes as blackfolds

Thermal spinning giant gravitons and null waves

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with J. Armas, A. Vigand Pedersen

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(with R. Emparan, T. Harmark, V. Niarchos)

Plan

- Introduction & blackfold approach
  - closed string (SUGRA) description of thermal probes
- Heating up DBI/NG solutions using:
  blackfolds as thermal probe branes/strings in string theory

  3 applications (new qualitative & quantitative effects)

- Thermal Bion solutions (wormhole & spike)
- Thermal string probes in AdS & finite T Wilson loops
- Thermal (spinning) giant gravitons
- Conclusion & outlook

Introduction

branes/strings have many applications:

- gauge theory: low energy dynamics, flavors, geometric picture of FT dualities
- quantum gravity: microstates of BHs, information paradox
- gauge/gravity (holography)

■ Brane/string probes widely used in ST, including AdS/CFT

- uncover features of backgrounds, phase transitions, stringy observables, non-perturbative aspects of FT, dual operators in CFT, ads/CMT
- learn new things about fundamentals of ST/M-theory by studying low energy theories on D/M-branes

conventionally used (at weak coupling):

F-stings: NG (Wilson loops, q-qbar potential, energy loss of quarks)
D-branes: DBI (Wilson loops in large sym/antisym reps, flavors, meson spectroscopy, giant gravitons)
M-branes: PST (giant gravitons, self-dual string)

Open/closed perspectives

worldvolume (DBI,NG)
microscopic, open
weak coupling

⟷

spacetime (SUGRA)
macroscopic, closed
strong coupling

- for SUSY configs can interpolate between the two (exactly)
- for non-SUSY (finite T): qualitative matching (more control for near-extremal)

"shapes of branes/strings" are determined dynamically

most work on curved D-branes performed using open string picture:

probe (N=1)

⇨ $$K_{ab}{}^{\rho} T^{ab} = J \cdot F^{\rho}$$

can use symmetries, ansatze consistency to construct the exact backgrounds in SUGRA (N >> 1)

extrinsic curvature (2nd fundamental form)

(DBI) EM tensor of the brane

external force

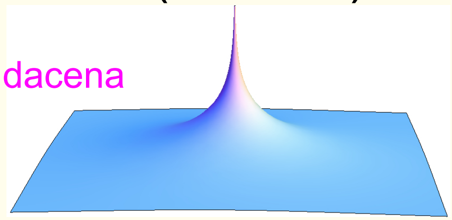
Note: any probe brane will have EOM of this form Carter

Example: BIon

DBI

D3-brane DBI with constant electric flux (D3->F1)

Callan,Maldacena



SUGRA

derive appropriate PDEs and prove existence of SUSY sol

Lunin

branes follow harmonic profiles (unique, given BCs)

match: open/closed duality beyond decoupling limit

Q: Can one extend this open/closed picture to finite T ? (non-SUSY)

interesting since:
- develop horizon, learn about BH physics
- branes are used to probe spaces at finite T (hot flat or AdS space, AdS BH)
- thermal states in gauge theories (AdS/CFT)

Intermezzo: conventional method for probe branes in thermal background

conventionally used method: 'Euclidean DBI probe' method:
- Wick rotate background and classical DBI action
- find solns. of EOM
- identify the radii of thermal circle in background and DBI soln.

(see also: Kiritsis/Kiritsis,Taylor/Kiritsis,Kehagias)

boils down to: solving same (local) EOMs but different BCs

$$(T_E)^{ab}_{\rm DBI}(K_E)_{ab}{}^{\rho} = (\perp_E)^{\rho\lambda}\frac{1}{4!}(J_E)^{abcd}(F_E)_{\lambda abcd}$$

this global condition is not enough to ensure that probe is in thermal equilibrium with the background

reason: to ensure thermal equilibrium we need to also modify the EOMs (via the stress tensor) since the brane DOFs get thermally excited

Example: single D3-brane near extremality (at weak coupling)
- gas of photons (+ superpartners)

$$T_{ab} = -T_{\rm D3}\eta_{ab} + T^{\rm (NE)}_{ab}\ , \quad T^{\rm (NE)}_{00} = \rho\ , \quad T^{\rm (NE)}_{ii} = p\ , \ i=1,2,3 \qquad \rho = 3p = \pi^2 T^4/2.$$

Open/closed at finite T

open (weak coupling) , N=1

thermal DBI
(thermal SUSY gauge theory
+ string corrections)

Grignani,Harmark,Marini,Orselli (to appear)

thermal NG: quantize string in finite T background

de Boer,Hubeny,Rangamani,Shigemori

closed (strong coupling), N >>1

black branes (solitons)
- curved black brane solutions in SUGRA

exact solutions already hard at T=0

go to regime where brane is approximately **locally** flat:
-> can use probe approximation
= 0^{th} order blackfold construction

gives the geometry to leading order in perturbative expansion governed r0/R

like DBI/NG this is (to leading order) probe computation:
dynamics in both cases described by Carter equation:

- difference is EM tensor that you put
  + different regime ! (match when T->0,N=1)

Blackfold approach

novel treatment of SUGRA solutions:

- uses effective (long-wave length) expansion scheme (technically/conceptually closer to worldvolume description)
- provides immediate intuitive information and easy access to thermodynamical quantities
- extends to more complicated (less symmetric) configurations that are beyond reach of current exact solution generating techniques

provides effective description of black brane dynamics in terms of fluid living on dynamical worldvolume (describes how it fluctuates, spins, bends,..)
roughly: mix of fluid dynamics (cf. fluid/gravity) + “DBI”

- effective degrees of freedom: slowly-varying energy density+ fluid velocities + world volume bending scalars + charge/spin densities etc.

⇨ $X^\mu(\sigma), \varepsilon(\sigma), u^\mu(\sigma), q(\sigma), \ldots$

- EOMs for these follow from conservation of stress-energy tensor, charge currents, …

$$\overline{\nabla}_\mu T^{\mu\rho} = 0$$

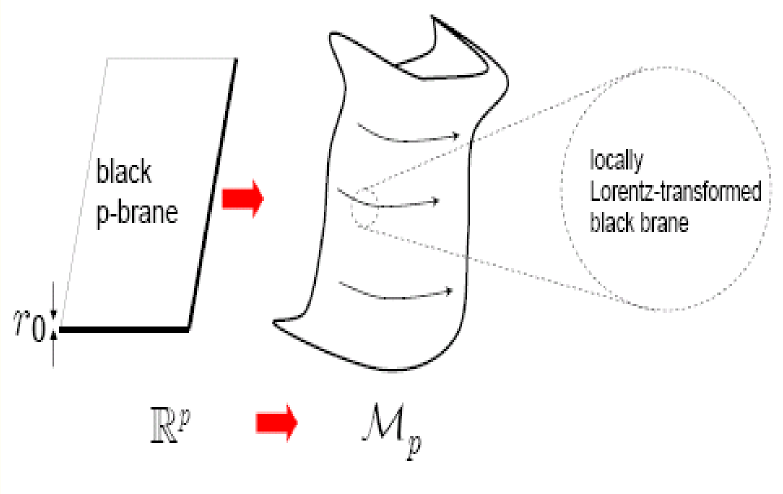
- constructive procedure that maps solutions to regular bulk spacetimes
- but already interesting results at leading order

Blackfold equations

Emparan,Harmark,Niarchos,NO

take black branes (possibly charged, intersections/bound states) and curve them (e.g. into black holes with compact horizon topologies)

integrate out brane thickness
➔ effective description involving fluid living on the brane



size-scale of the brane << length scale of the worldvolume W_{p+1}

blackfold equations

- intrinsic equations:
  conservation of EM on the w.v.
  & charge conservation

$$D_a T^{ab} = 0 \qquad D_a J^{aa_1 \ldots a_p} = 0$$

- extrinsic equations: $K_{ab}{}^{\rho} T^{ab} = J \cdot F^{\rho}$

same form as DBI/NG !

generalization using charged branes:

Emparan,Harmark,Niarchos,NO
Caldarelli,Emparan,v.Pol
Grignani,Harmark,Marini,NO,Orselli

use EM tensor of black brane instead of DBI EM tensor

Stationary blackfolds and action principle

powerful action principle to arrive at effective action, yielding BF EOMS:

$$I = \int_{W_{p+1}} \sqrt{-\gamma} P$$

For stationary (i.e. also static) configurations:
extrinsic equations for the embedding can be integrated to an action
= Gibbs free energy

$$\beta I_E = F = M - TS - \Omega J$$

$$M = \int dV_{(p)} T_{00}$$

$$S = \int dV_{(p)} s$$

$$J = \int dV_{(p)} T_{0\phi}$$

extrimizing this action for fixed temperature, angular velocity, charge is equivalent to 1st law of thermo

1st law of thermo = blackfold equations for stationary configurations

effective action = "thermal version" of DBI/NG:
but in supergravity (closed string) regime

Applications

-> Heating up DBI/NG solutions using:
blackfolds as thermal probe branes/strings in string theory

shows new qualitative & quantitative effects

- Thermal Bion solutions (wormhole & spike)
  (briefly also: M5-M2 system and self-dual string)

- Thermal string probes in AdS &
  finite T Wilson loops

- Thermal (spinning) giant gravitons

black probe	background
D3-F1	hot flat 10D
F1	AdS BH
	hot..
D3	AdS5 x S5
M2	AdS7 x S4
M5	AdS4 x S7

“Recipe” for stationary blackfold solutions

- find energy momentum tensor (+ charge current) of (charged) black brane configuration one wants to bend
  (perfect fluid with some specific equation of state)
  How: use flat black brane (SUGRA) solution
  + determine stress tensor/current that sources it

- describe the geometric setup (background + embedding)

- let the collective modes fluctuate over the brane worldvolume, given the geometric setup
  + impose global temperature, angular velocities + charges constant.

- write down the thermodynamic action (off-shell functional)

- vary action to get equations of motion + solve for embedding

- given solution compute the on-shell thermodynamic quantities + analyze

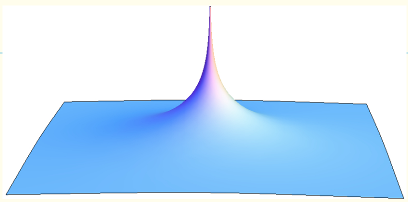
BIon solution of DBI

DBI describes dynamics of U(1) gauge field living on D-brane + scalars describing transverse fluctuations

➔ D3-brane with constant w.v. flux F_{01}.

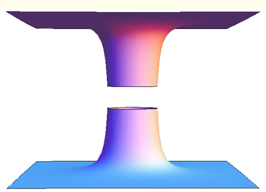
use DBI EOM: embedding profile of D3-brane

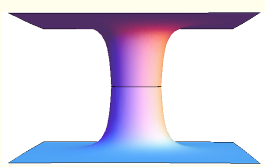
F-strings dissolving into D-brane



- k F-strings ending on N coincident infinitely extended D3-brane (spike)

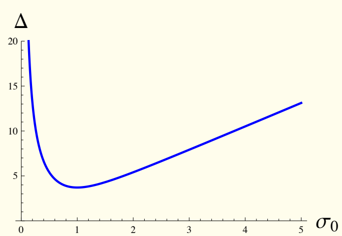
$$z(\sigma) = \frac{\kappa}{\sigma} \qquad \left.\frac{dH}{dz}\right|_{\sigma=\sigma_0=0} = 4\pi\, T_{\rm D3}\kappa = kT_{\rm F1}$$





k F-strings stretching between parallel system (wormhole): finite throat size

$$z(\sigma) = \int_\sigma^\infty d\sigma' \frac{\sqrt{\sigma_0^4+\kappa^2}}{\sqrt{\sigma'^4-\sigma_0^4}}$$



Action for thermal Bion + solution

Use black D3-F1 brane SUGRA soltion to get stress tensor/currents
action for thermal Bion takes DBI-like form:

$$\mathcal{F}(T,N,k) = \frac{2T_{D3}^2}{\pi T^4}\int_{\sigma_0}^{\infty} d\sigma \sqrt{1+z'(\sigma)^2}F(\sigma)$$

$$F(\sigma) = \sigma^2 \frac{1+4\sinh^2\alpha(\sigma)}{\cosh^4\alpha(\sigma)}$$

relation to DBI:

$$\lim_{T\to 0}\mathcal{F} = NH_{\mathrm{DBI}}$$

EOM can be integrated exactly

$$z'(\sigma) = -\left(\frac{F(\sigma)^2}{F(\sigma_0)^2} - 1\right)^{-\frac{1}{2}}$$

reproduces extremal BIon in zero temperature limit

$$-z'(\sigma) = \sqrt{\frac{\kappa^2+\sigma_0^4}{\sigma^4-\sigma_0^4}}\left[1+\mathcal{O}(\bar{T}^4)\right] \qquad \sim \frac{\kappa}{\sigma^2} \quad (\text{for } \sigma_0 = 0)$$

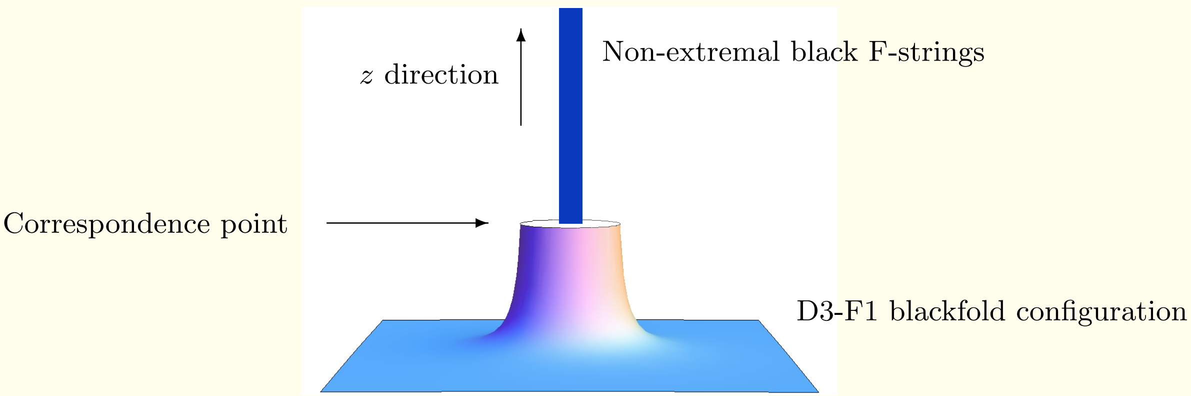
validity of the probe approximation: $\sigma_0^3 \gg \sqrt{k}g_s l_s^3$

Thermal spike ?

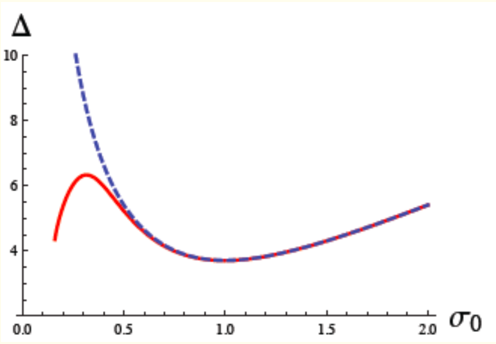
thermal Bion BF solution does not allow for infinite spike

but: configuration in question can be made by matching up a non-extremal black F-string solution with throat solution !

non-trivial: two independent gluing conditions, tension & entropy density agree impressively good



- Thermal wormhole Bion solutions studied in detail: three phases for given brane separation instead of two



Generalization to M2-M5 funnel

Niarchos,Siampos

apply to M2-M5 brane system (D1-F3 cousin): fully localized intersection
¼ BPS intersection described by 3-funnel (spike) solution of effective 5-brane worldvolume theory

$$z(\sigma) = 2\pi \frac{N_2}{N_5} \frac{\ell_P^3}{\sigma^2}$$

self-dual string soliton in new regime

also: thermal version using BF (wormhole & "spike")

find: $$c \simeq 0.6 \frac{N_2^2}{N_5}$$

prediction for central charge of self-dual string, valid for large N2, N5

use again non-trivial gluing conditions at correspondence point & extrapolation

from dimensional considerations

$$\frac{1}{\lambda} := \frac{q_2}{q_5^2} = \frac{4N_2}{N_5^2}$$

-> implies the scaling

$$c \sim 0.04 \frac{N_5^3}{\lambda^2} \qquad c \sim 0.3 \frac{N_2^{\frac{3}{2}}}{\sqrt{\lambda}}$$

Thermal string probes in AdS

apply BF description of thermal string probes to finite T Wilson loops in context of AdS/CFT

- previous studies: use extremal (Nambu-Goto) probe in AdS BH background

Brandhuber,Itzhaki,Sonnenschein,Yankielowicz/Rey,Theisen,Yeei

classical NG action becomes increasingly inaccurate since as string approaches event horizon, local temperature for static string probe is redshifted towards infinity

- want to take into account thermal degrees of freedom for more accurate description (and see what effects are)

- one way: quantize NG action in finite T background to include thermalization leading quadratic correction for string probing AdS BH

$k = 1,\ g_s \ll 1$

de Boer,Hubeny,Rangamani,Shigemori

- other approach (BF): use SUGRA solution for non-extremal F-string to describe finite T Wilson loop

SUGRA F-string requires many (k) coincident F-strings:
so describes Wilson loop in k-symmetric product of fundamentals
regime of validity: $1 \ll k \ll N,\ g_s^2 k \gg 1.$

Set up

consider rectangular Wilson loops in finite T N=4 SU(N) SYM in large N limit

expectation value of WL gives potential for Q-Qbar, Q = kth symmetric rep of k quarks

background: black hole in AdS5 (Poincare patch)

Q-Qbar pair dual to k coincident probe F-strings attached to locations of the two particles on the boundary (distance L)

reveals:

- new correction term, in small LT expansion of free energy, in Coulomb potential (as compared to first correction of extremal F-string probe), can become leading correction for sufficiently small termperatures

- finite LT: phase transition to phase with two Polyakov loops (one for each charge) -> Debye screening of charges
order 1/N correction to onset of Debye screening relative to extremal F-string

- careful analysis of validity of probe approximation: breaks down close to event horizon

relevant configurations

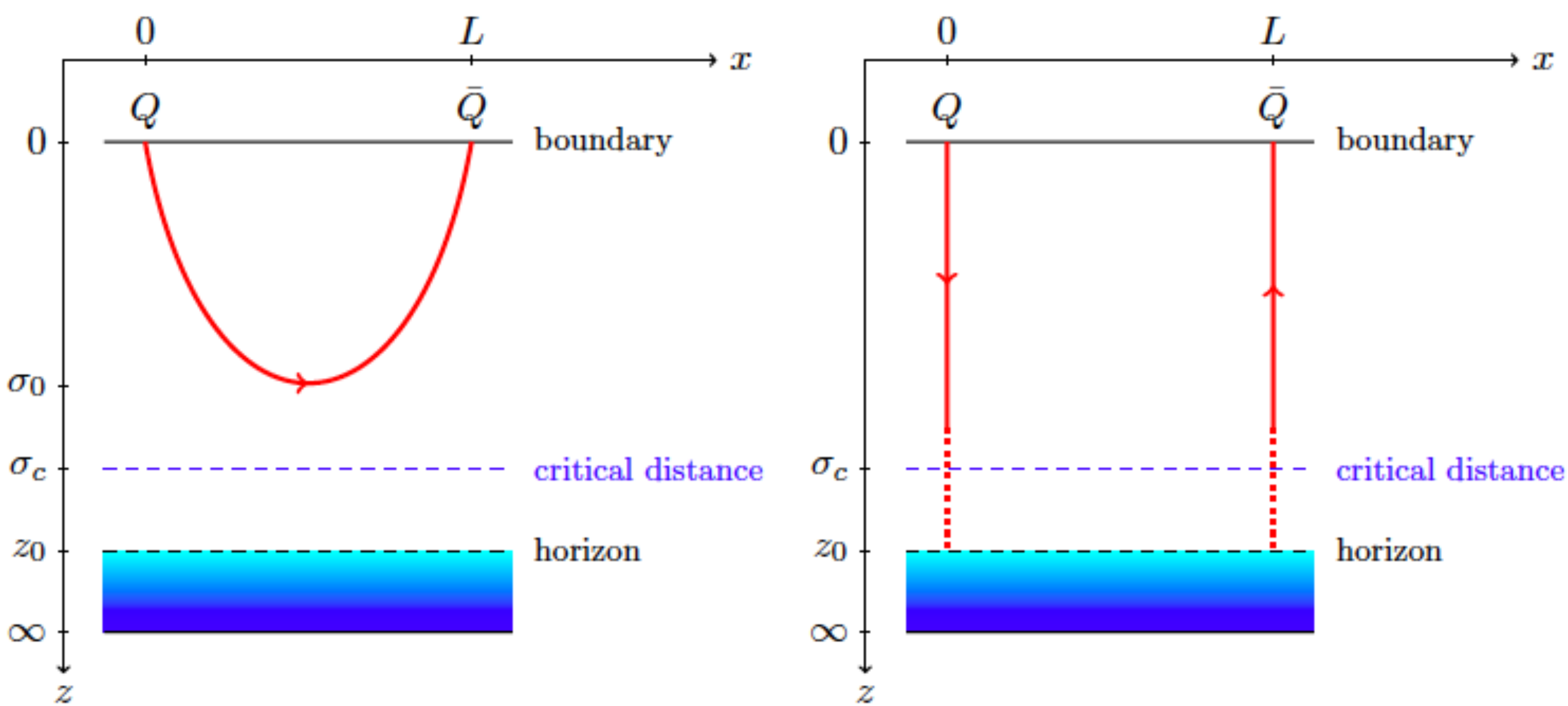
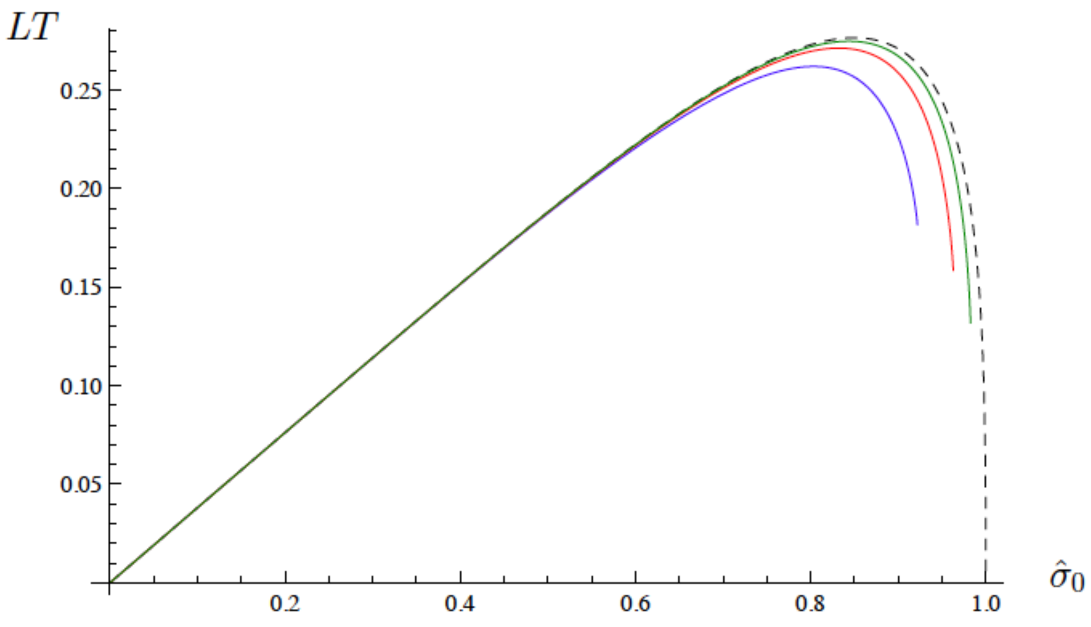


Figure 1. Left side: F-strings stretched between the Q-$\bar{Q}$ pair corresponding to a rectangular Wilson loop. Right side: Two straight strings stretching from the two charges to the event horizon corresponding to two Polyakov loops.

find critical distance to event horizon beyond which the probe cannot go
(consequence of thermal equilibirium & fact that non-extremal F-strings have a max T)

finite LT results



dashed line: NG result

$$LT|_{\kappa=0} = \frac{2\sqrt{2\pi}}{\Gamma\left(\frac{1}{4}\right)^2}\hat{\sigma}_0\sqrt{1-\hat{\sigma}_0^4}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{5}{4};\hat{\sigma}_0^4\right)$$

$$\kappa = \frac{2^7}{3^6}\frac{k\sqrt{\lambda}}{N^2} \qquad \hat{\sigma}_0 = \pi T \sigma_0$$

$$LT = \frac{2\sqrt{2\pi}}{\Gamma(\frac{1}{4})^2}\hat{\sigma}_0 + \left(\frac{\sqrt{2\pi}}{3\Gamma(\frac{1}{4})^2} - \frac{1}{6}\right)\sqrt{\kappa}\hat{\sigma}_0^4 - \frac{2\sqrt{2\pi}}{5\Gamma(\frac{1}{4})^2}\hat{\sigma}_0^5 + \mathcal{O}(\hat{\sigma}_0^7)$$

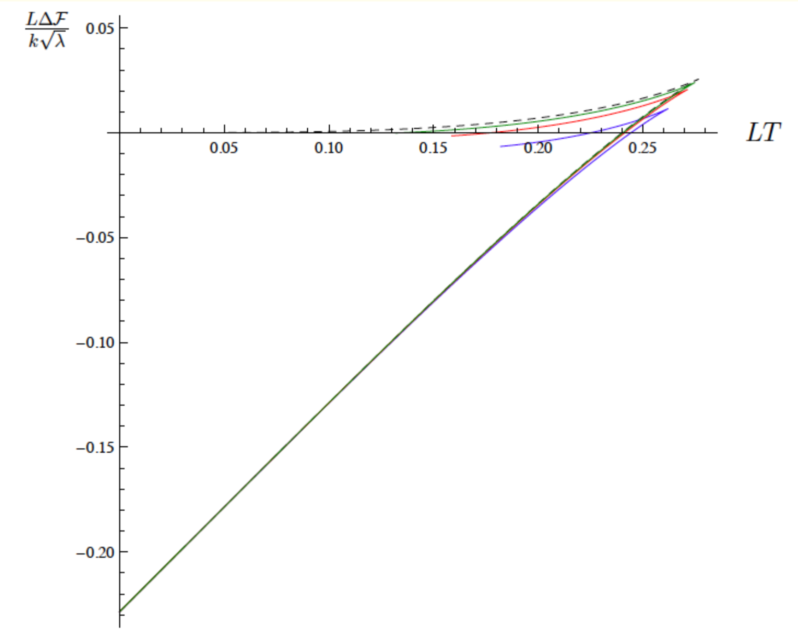
$$\mathcal{F}_{\text{loop}} = -\frac{\sqrt{\lambda}k}{L}\left(\frac{4\pi^2}{\Gamma(\frac{1}{4})^4} + \frac{\Gamma(\frac{1}{4})^4}{96}\sqrt{\kappa}(LT)^3 + \frac{3\Gamma(\frac{1}{4})^4}{160}(LT)^4 + \cdots\right)$$

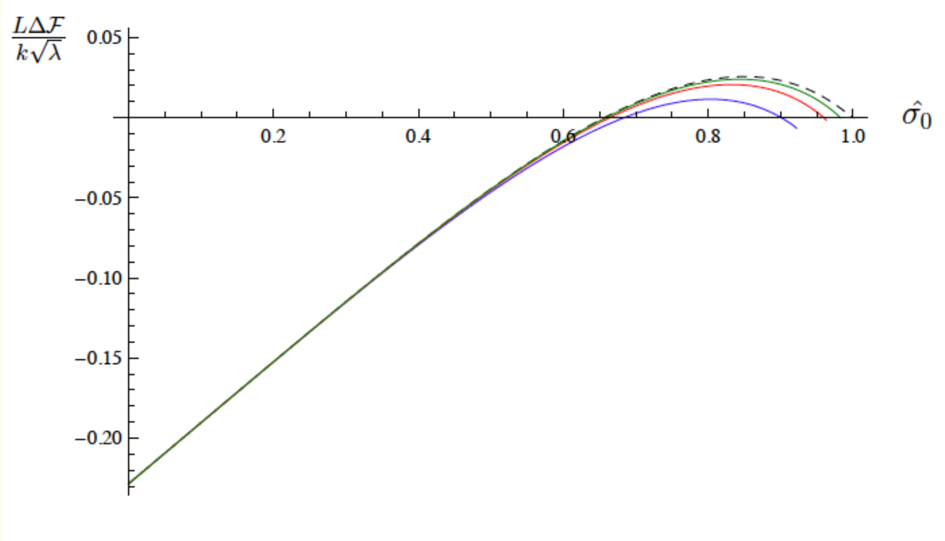
thermal probe correction

extremal (NG) probe

⇨ dominant for $LT \ll \frac{\sqrt{k}\lambda^{1/4}}{N}$

Debye screening





$$\Delta\mathcal{F} = \mathcal{F}_{\rm loop} - 2\mathcal{F}_{\rm charge}$$

$$\begin{aligned}\Delta\mathcal{F}|_{\kappa=0} &= \sqrt{\lambda}kT\left[-\frac{1}{\hat{\sigma}_0}+1+\int_0^{\hat{\sigma}_0} d\hat{\sigma}\frac{1}{\hat{\sigma}}\left(\hat{\sigma}_0^2\sqrt{\frac{\hat{\sigma}^4-1}{\hat{\sigma}^4-\hat{\sigma}_0^4}}-1\right)\right] \\ &= \sqrt{\lambda}kT\left[1-\frac{\sqrt{2}\pi^{3/2}\left(1-\hat{\sigma}_0^4\right)}{\hat{\sigma}_0\Gamma\left(\frac{1}{4}\right)^2}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{1}{4};\hat{\sigma}_0^4\right)\right],\end{aligned}$$

$$(LT)|_{\Delta\mathcal{F}=0} \simeq 0.240038 + 0.0379706\sqrt{\kappa}$$

onset of screening increases
(slightly less easily screened)

Thermal giant gravitons

Armas,Harmark,NO,Orselli,Vigand-Pedersen

apply thermal probe-brane method to giant graviton (GG)
- archetypical case: D3 wrapped on three-sphere and with CM moving along five-sphere (blown up version of point particle graviton, uses DBI)

dual gauge theory: GG moving with ang. mom. J dual to multi-trace op. $\mathcal{O}_{\rm gg}$ with R-charge J and $\Delta = J$

heating up: thermal state resulting from ensemble of ops. that are fluctuations around $\mathcal{O}_{\rm gg}$

true up to $T_{\rm HP}$: AdS BH is formed

find following features for thermal GG

- solutions have J > J_min (cf. extremal where J -> 0 for point particle) thermal GG is forced to blow up (cf Bion, which has minimal radius)
- we find T_max, sets scale of soln, but probe requires T_max >> T_HP so if we should be well below HP, we have T/T_max << 1 (cf thermal rectangular Wilson loop)
- free energy in low T limit:

$$F(T,J) = \frac{J}{L} - \frac{\pi^4}{4} N_{\rm D3}^2 L^3 T^4 + \mathcal{O}(T^8)$$

- for J_min < J < J_max: two available solutions (stable +unstable) (cf Bion +WL, where number of solutions changes)
- also: new stable branch for the extremal GG

Extremal GG in AdS5xS5

parameterize S5 as

$$\mathrm{d}\Omega_{(5)}^2 = L^2\left[\mathrm{d}\zeta^2 + \cos^2\zeta d\phi_1^2 + \sin\zeta^2 \mathrm{d}\Omega_{(3)}^2\right]$$

embedding of D3

$$t = \tau, \quad \phi_1 = \Omega\tau, \quad \phi_2 = \sigma_1, \quad \phi_3 = \sigma_2, \quad \theta = \sigma_3\ , \quad \zeta = \mathrm{const.}$$

induced metric on D3-brane

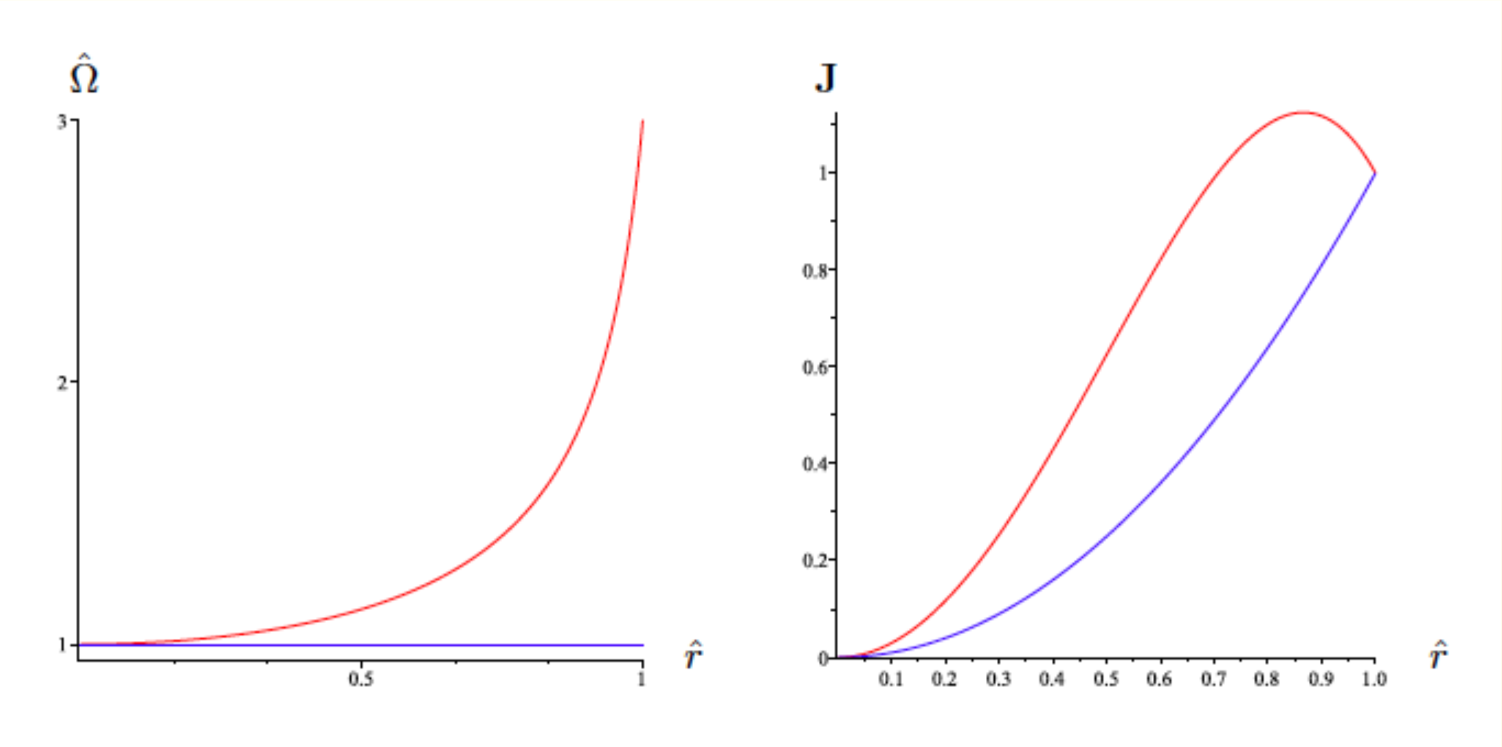
$$\gamma_{ab} d\sigma^a d\sigma^b = -\mathrm{k}^2 d\tau^2 + r^2 d\Omega_{(3)}^2$$

$$\mathrm{k} \equiv |k| = \sqrt{1 - \Omega^2(L^2 - r^2)}$$

DBI action

$$L_{\mathrm{DBI}} = -T_{\mathrm{D3}}\Omega_{(3)} r^3 \left(\mathrm{k} - r\Omega\right)$$

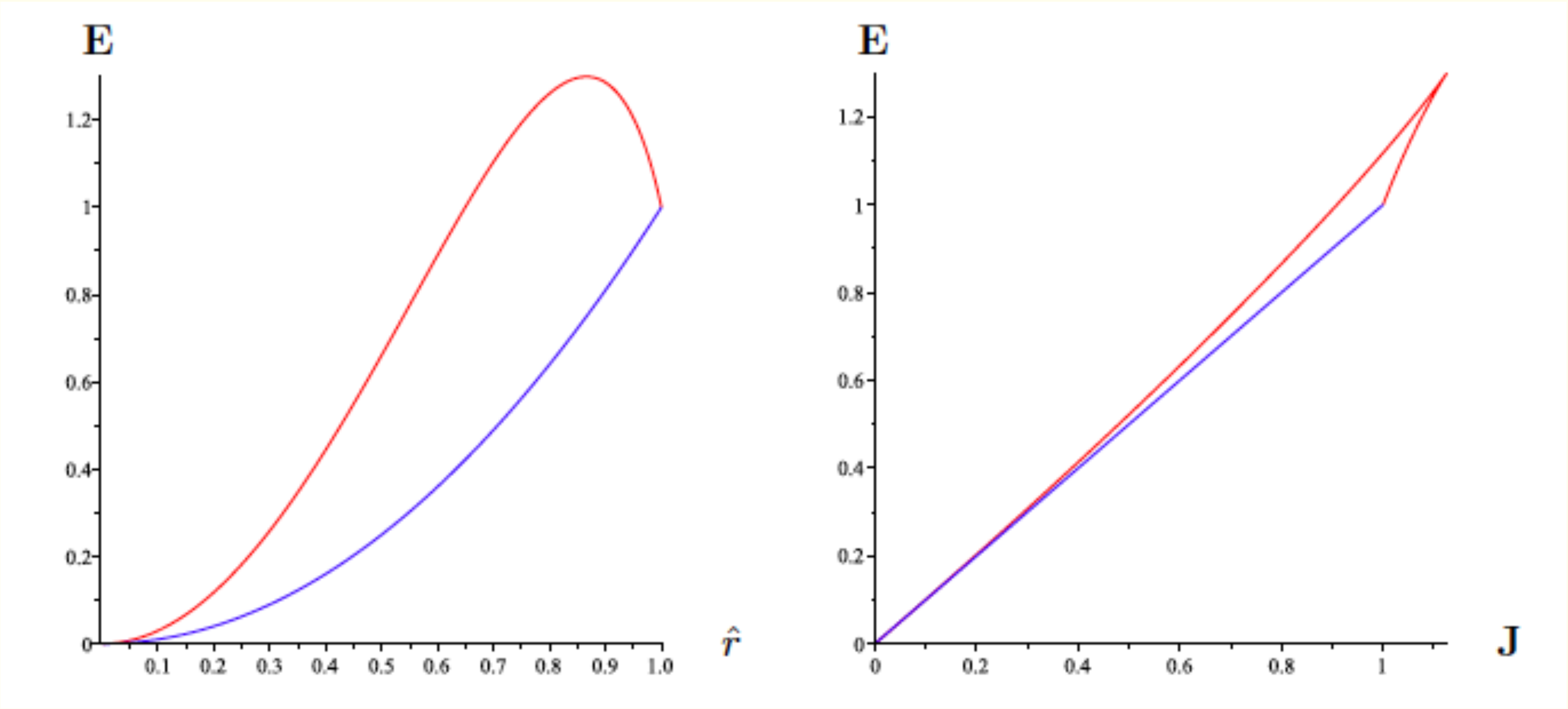
Extremal GG solutions



lower branch: usual BPS branch
upper branch: non-BPS branch (1/2 BPS in limit r=L)
stable for sqrt(3)/2 < r/L < 1

- two solutions for $0 < J < 9N/8$ (one stable, one unstable)

Extremal GG: energy & stability



lower (blue) branch for 0 <= J <= 1 always stable (conventional)
upper (red) branch has stable part for 1 <= J <= 9/8

Heating up the GG

need to generalize BF approach slightly

- BF EOM for charged branes in backgrounds with fluxes
  (also: extra contributions to conserved quantities, action etc.)

- brane probe moves with constant velocity along Killing direction,
  (not stationary solution), but not accelerating since moving along geodesic
  we call this quasi-stationary blackfolds (boosted stationary BFs)

cf. Camps,Emparan,Figueras,Giusto,Saxena

* does not emit radiation/can go beyond probe using MAE

interpretation of conserved quantities:
properties of the BF probe moving in the background
(not of probe +background
cf energy + momentum of particle moving with constant velocity)

BF in flux backgrounds

BF EOM

$$T^{ab}K_{ab}{}^{\mu} = \frac{1}{(p+1)!}\perp^{\mu}{}_{\nu}F^{\nu\rho_1\cdots\rho_{p+1}}J_{\rho_1\cdots\rho_{p+1}}$$

conserved quantities

$$E = \int_{\mathcal{B}_p} dV_{(p)}\gamma_{\perp}^{-1}\left[T^{\mu\nu} + \mathcal{V}^{\mu\nu}\right]n_{\mu}\xi_{\nu}\ , \quad J = -\int_{\mathcal{B}_p} dV_{(p)}\gamma_{\perp}^{-1}\left[T^{\mu\nu} + \mathcal{V}^{\mu\nu}\right]n_{\mu}\chi_{\nu}$$

$$\mathcal{V}^{\mu\nu} \equiv \frac{1}{p!}A^{\nu}{}_{\mu_1\cdots\mu_{p-1}}J^{\mu\mu_1\cdots\mu_{p-1}}$$

$\gamma_{\perp}$ transverse Lorentz contraction factor

mechanical action

$$I = \int_{\mathcal{W}_{p+1}}\left\{\omega_{(p+1)}P + Q_p\mathcal{P}[A_{(p+1)}]\right\}$$

extra "WZ" term

= thermodynamic action

$$\frac{I_E}{\beta} = F = E - \Omega J - TS$$

finite T Giant Graviton

use data of black D3-brane $T_{ab} = (\epsilon + P) u_a u_b + P \gamma_{ab}$

$$\epsilon = \mathcal{T} s - P \ , \quad P = -\mathcal{G} \left(1 + 4 \sinh^2 \alpha\right) \ , \quad \mathcal{T} s = 4\mathcal{G} \ , \quad \mathcal{G} \equiv \frac{\pi^2}{2} T_{\rm D3}^2 r_0^4$$

$$\mathcal{T} = \frac{1}{\pi r_0 \cosh \alpha} \ , \quad s = 2\pi^3 T_{\rm D3}^2 r_0^5 \cosh \alpha$$

$$J_{(4)} = Q d\tau d\sigma^1 d\sigma^2 d\sigma^3 \ , \quad Q = 4\mathcal{G} \sinh \alpha \cosh \alpha = N_{\rm D3} T_{\rm D3}$$

thermal GG EOM $\Omega^2 r^2 \left(1 - \mathcal{R}_1(\alpha)\right) + 3 \mathrm{k}^2 + 4 \mathrm{k} \Omega r \mathcal{R}_2(\alpha) = 0$

$$\mathcal{R}_1(\alpha) \equiv \frac{\mathcal{T} s}{P} = -\frac{4}{1 + 4 \sinh^2 \alpha} \quad \text{and} \quad \mathcal{R}_2(\alpha) \equiv \frac{Q}{P} = -\frac{4 \sinh \alpha \cosh \alpha}{1 + 4 \sinh^2 \alpha}$$

subject to charge conservation:

$$N_{\rm D3} T^4 = \frac{2 T_{\rm D3}}{\pi^2} \frac{\sinh \alpha}{\cosh^3 \alpha} \mathrm{k}^4$$

local temperature is redshifted:

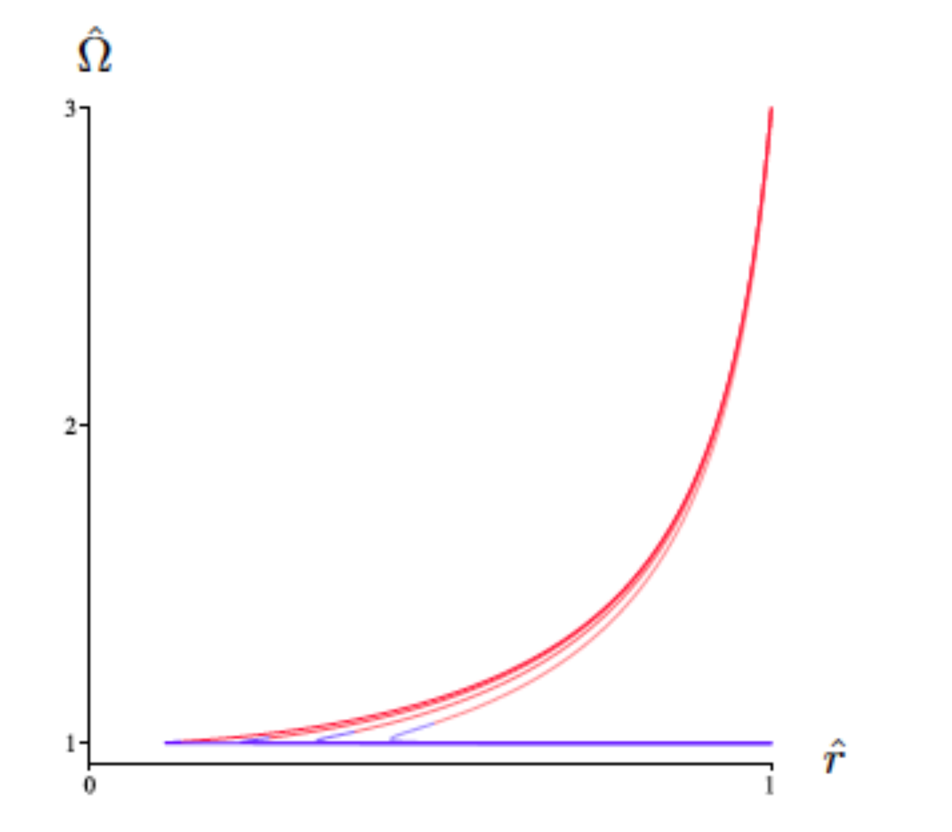
$$\mathcal{T} = T / \mathrm{k}$$

Thermal GG solution space

system has
max temperature

$$\hat{T} \equiv \frac{T}{T_{\rm max}}\ , \quad T_{\rm max} \equiv \left(\frac{8\sqrt{5}}{27\pi^2}\frac{T_{\rm D3}}{N_{\rm D3}}\right)^{1/4}$$

$$\hat{T} \leq \mathbf{k} \leq 1$$



- minimal possible size
  no point particle limit !

- upper/lower branch connected

interesting regimes & validity

- low temperatures: expand around extremal ase
- maximal size, expand around r=L (k=1)
- minimal charge parameter limit: expand around k=T, i.e. the point where the two branches meet

validity (D3-branes in sugra approximation)

$$N_{D3} \gg 1 \text{ and } N \ll \lambda N_{D3}$$

probe approx of BF

$$N_{D3} \ll N$$

validity:

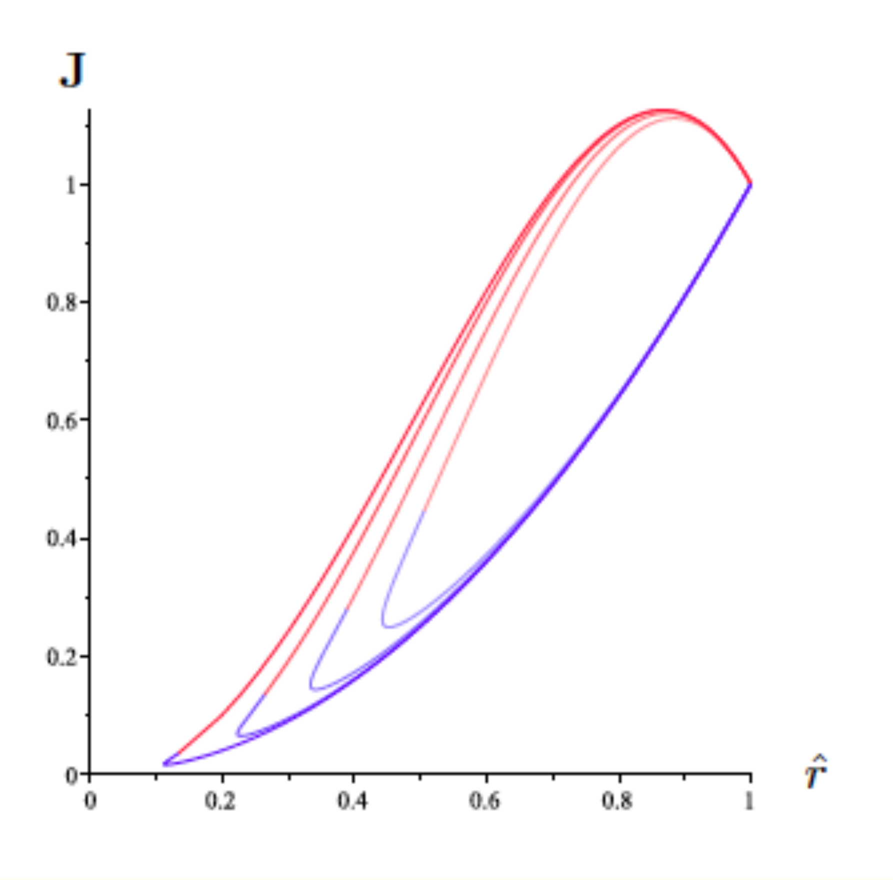
$$1 \ll N_{D3} \ll N \ll \lambda N_{D3}$$

$$\frac{T_{max}}{T_{HP}} \sim \left(\frac{N}{N_{D3}}\right)^{1/4} \gg 1$$

-> so need T << T_max,
otherwise in regime where AdS BH background dominates

action + angular momentum

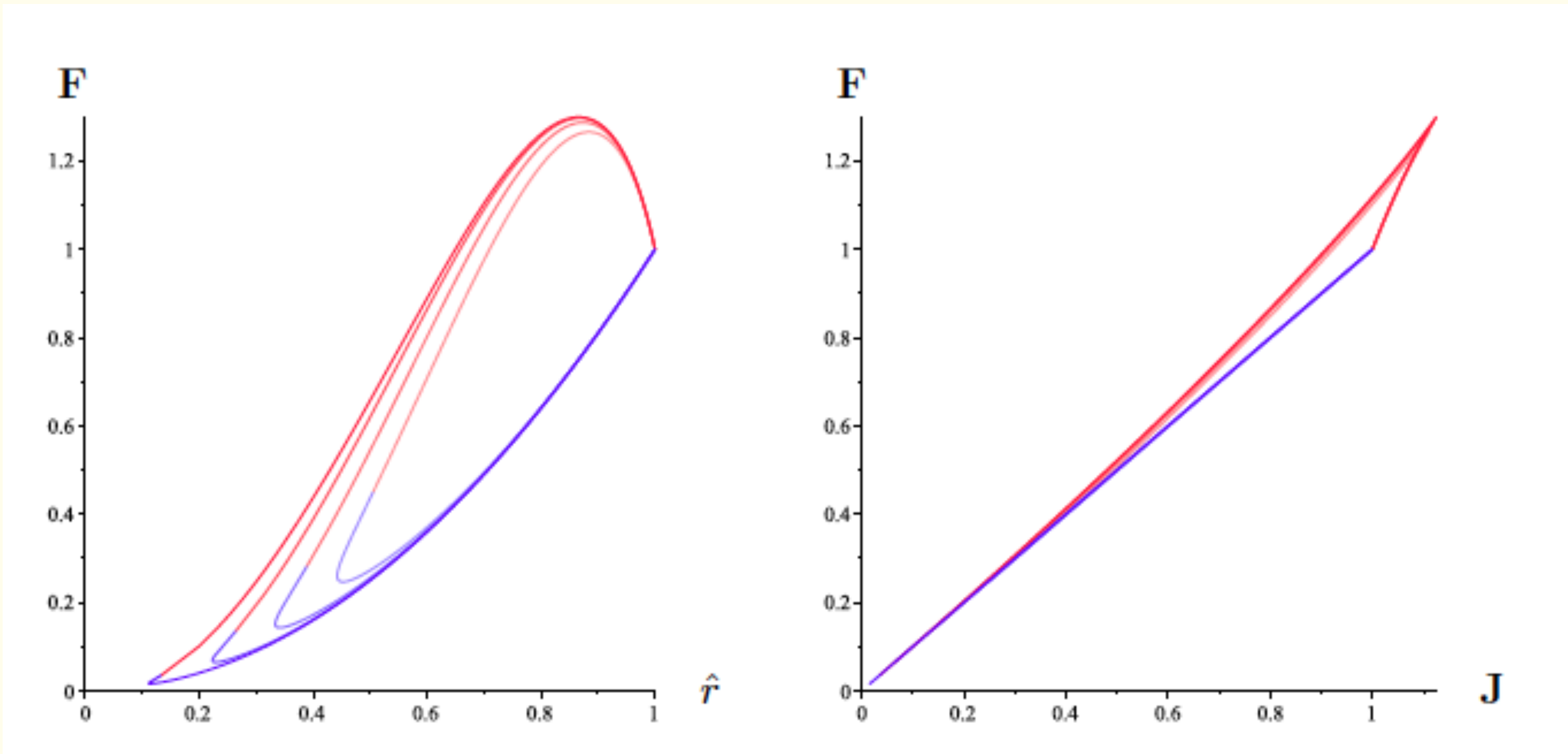
$$\beta I_E = F = E - TS - \Omega J = -\Omega_{(3)} \frac{T_{\rm D3}^2}{2}(r^3 \mathbf{k} P + r^4 \Omega Q)$$



$$J_{\rm min}(\hat{T}) \le J \le J_{\rm max}(\hat{T})$$

- less phase space as temperature increases

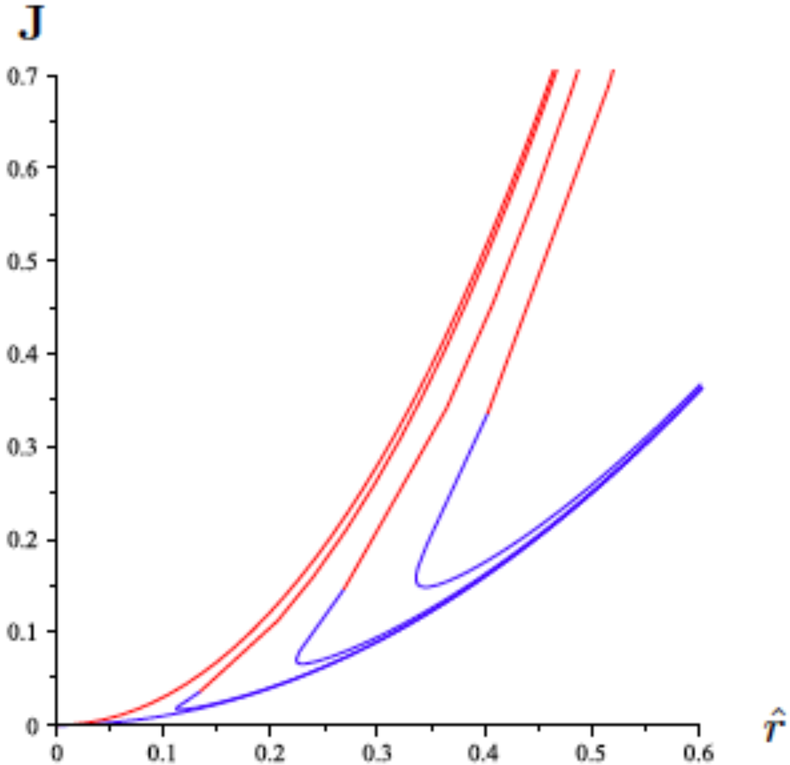
thermodynamics/stability

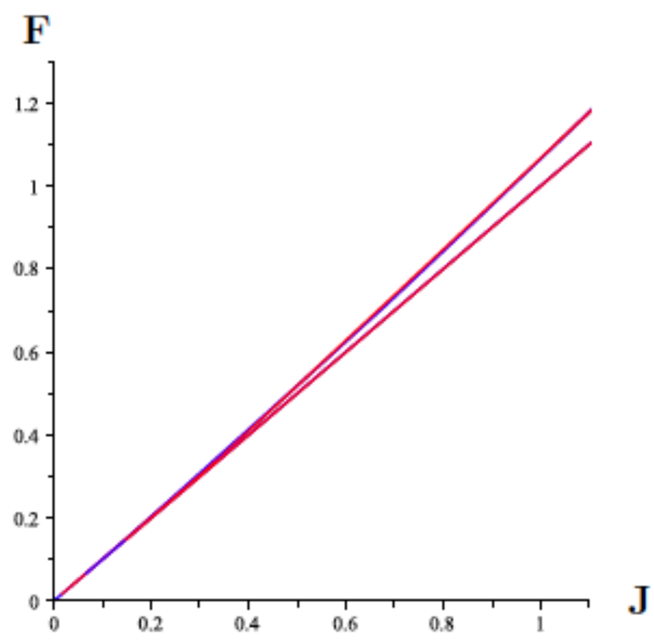


$$F(T,J) = \frac{J}{L} - \frac{\pi^4}{4} N_{\rm D3}^2 L^3 T^4 + \mathcal{O}(T^8)$$

prediction for dual
GT at strong coupling

thermal GG moving on AdS5





only (part of) lower branch
is stable

again:

$$F(T,J) = \frac{J}{L} - \frac{\pi^4}{4} N_{\mathrm{D3}}^2 L^3 T^4 + \mathcal{O}(T^8)$$

Spinning Giant Gravitons

Armas,NO,Vigand-Pedersen (to appear)

one can spin up the thermal giant graviton in the internal S3 directions
- not possible for the SUSY GG because of Lorentz invariance on the w.v.

$$k_F^a \partial_a = \partial_\tau + \omega \sum_j \partial_{\phi_j}$$

angular velocity w on the two U(1) directions in S3

interesting to consider effect of extra quantum number (spin S) on the phase space (EOM still solvable)

- maximum possible internal spin S (for given T)
- stable branch
- for maximum size GG: E \sim S

\+ new extremal limit describing a nullwave giant obtained by taking a double scaling limit:

$$\phi = (\cosh^2 \alpha)^{-1} \to 0, \ |\mathbf{k}| \to 0$$
$$\phi/|\mathbf{k}|^2 = \mathcal{P} = \text{fixed}$$

P = null momentum density

Low T spinning giants

free energy around the extremal GG $F = \frac{J}{N} + \Delta F \, , \quad \Delta F < 0$

D3 on S3 $\Delta F \sim \frac{1}{L} N_3^2 (LT)^4$ $S \propto T^4$

M5 on S5 $\Delta F \sim \frac{1}{L} N_5^3 (LT)^6$ $S \propto T^6$

M2 on S2 $\Delta F \sim \frac{1}{L} N_2^{\frac{3}{2}} (LT)^3$ cannot spin on S2

thermal states have free energies that are (up to numerical factors) those for the D3,M5,M2 field theories

Nullwave Giant

spectrum

$$\mathbf{E} = \frac{1}{\hat{\omega}}\left(1 + \mathcal{P}\hat{\omega}^2\hat{r}^2\right)\hat{r}^{n-3} \quad , \quad \mathbf{J} = \mathbf{E}\hat{\rho}\sqrt{1-\hat{\omega}^2\hat{r}^2} + \hat{r}^{n-1}$$

$$\mathcal{S} = \beta\mathcal{P}\hat{\omega}^2\hat{r}^{n+1} \quad , \quad \hat{T}\mathbf{S} = 0 \quad , \quad \beta = \frac{2}{(p+1)(m-1)}$$

$$\mathbf{E} = \mathbf{J} + \frac{\mathcal{S}}{\beta}$$

for maximum size GG: $\mathbf{E} = \mathbf{J} + \mathcal{P}$

- new extremal solution: not SUSY + not captured by DBI
- action $I[X^\mu] = -Q_p \int_{\mathcal{W}_{p+1}} \sqrt{-\gamma}\left(1 + \frac{1}{2}\mathcal{P}k_a k^a\right)$
- EM tensor $T^{ab} = Q_p\mathcal{P}l^a l^b - Q_p\gamma^{ab}$

- zero temperature excitation of the ground state (in the closed string regime)
- open string counterpart: presumably need non-abelian DBI
  (but perhaps related to EM waves on GG: ->
  open-closed duality between electromagnetic and mechanical waves)

Summary & Outlook

- proposed a new method for F-string/D-brane probes in thermal backgrounds

- (can be used for all types of brane probes in thermal backgrounds
  (M-brane, NS5-brane) -> based on blackfold approach

+ applied to three cases:
(Bion in hot flat space, F-string in AdS BH, thermal GG)

discussed relation of this method to previous work:
- takes into account that the probe itself is a thermal object

generalize hot Bion to Dp-F1: qualitatively different features ? (cf M2-M5)

apply new perspective to AdS probes (thermal AdS or AdS BH)
- may resolve discrepancies between gravity and gauge theory
  found for Polaykov loops based on D3 (sym)/D5(antisym)
- revisit other previously studied cases

F-string in AdS: generalize to heavy quarks (BCs for string close to bdr)
energy loss of heavy quark moving thru plasma

examine correction term in quark potential at weak ‘t Hooft coupling

Summary & Outlook (cont'd)

- for GG find description of thermal state in dual gauge theory (compute free energy and compare)

- examine what happens above HP temperature (AdS BH bgr.)

- many GG moving along equator of S5 (smeared along circle)
  horizon topology change (when horizons overlap)
  study difference between smeared/non-smeared phases
  connection to superstar, LLM ½ BPS bubbling spaces (+ finite T generalization)

- further examine new extremal GG with nullwave
- heat up less SUSY GG (1/4, 1/8)

more generally:

- use blackfold formalism to:
  - go beyond probe level (backreaction effects)
  - study time evolution and stability

Camps,Emparan,Haddad
Armas,Camps,Harmark,NO
Camps,Emparan
Armas,Gath,NO

find first principles derivation from ST of the action describing thermal D-brane probes

- go beyond tree-level: effect of one or higher string loops

Relevance of BF method

EHONR/EHON/ Caldarelli,Emparan,Rodriguez
Armas,NO/Camps,Emparan,Giusto,Saxena/..

- new stationary BH solutions:

approximate analytic construction of BH metrics in higher D gravity/ supergravities (cf. String Theory)

- possible horizon topologies, thermodynamics, phase structure, …
- new non-extremal and extremal BH solutions
- useful for insights/checks on exact analytic/numeric solutions

Camps,Emparan,Haddad Armas,Camps,Harmark,NO
Camps,Emparan/Armas,Gath,NO/Armas,NO

- BH instabilities and response coefficients:

understand GL instabilities in long wavelength regime, dispersion relation, elastic (in) stabilities, new long wavelength response coefficients for BHs, Young modulus (hydro + material science)

GHMOO

- Thermal probe branes/strings:

new method to probe finite T backgrounds with probes that are in thermal equilibrium with the background (e.g. hot flat space, BHs)

- AdS/CFT:

many potential applications
(new black objects in AdS, connection with fluid/gravity, thermal probes thermal giant gravitons, BHs on branes, …)

+ interrelations between the four items above

The end

More on DBI-action and EOM

DBI action (restrict to D3 for simplicity)

$$I_{\rm DBI} = -T_{\rm D3}\int_{\rm w.v.} d^4\sigma\sqrt{-\det(\gamma_{ab}+2\pi l_s^2 F_{ab})} + T_{\rm D3}\int_{\rm w.v.} P[C_{(4)}]$$

EOMs by varying wrt embedding map

$$T^{ab}K_{ab}{}^{\rho} = \perp^{\rho\lambda}\frac{1}{4!}J^{abcd}F_{\lambda abcd}$$

with

$$T^{ab} = -\frac{T_{\rm D3}}{2}\frac{\sqrt{-\det(\gamma+2\pi l_s^2 F)}}{\sqrt{\gamma}}\left[((\gamma+2\pi l_s^2 F)^{-1})^{ab} + ((\gamma+2\pi l_s^2 F)^{-1})^{ba}\right]$$

$$J^{abcd} = T_{\rm D3}\frac{1}{\sqrt{\gamma}}\epsilon^{abcd}$$

BIon solution: stress tensor = extremal D3-F1 sugra solution (due to SUSY)

heating up the Bion: take stress tensor of non-extremal D3-F1 sugra solution

Example: BIon

■ DBI = low energy effective action for D-brane dynamics (integrating out massive open d.o.f.):

◆ 1st example that exploited full non-linear dynamics: BIon solution

new phenomena:

- multiple coincident F-strings described in terms of D-branes
- 1D F-string with zero thickness is blown up into higher-dim brane wrapped on sphere

➔ many important applications of DBI in ST & AdS/CFT (giant gravitons, blown up Wilson loops as D3 or D5 branes, …)

likewise: NG for string probes (Wilson loops, ..)

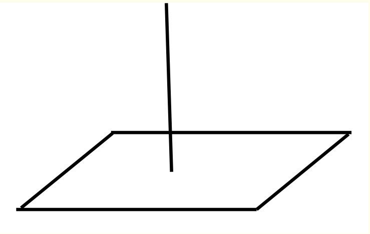
BIon solution of DBI

DBI describes dynamics of U(1) gauge field living on D-brane + scalars describing transverse fluctuations

linearized regime: Maxwell for gauge field + free, massless scalars

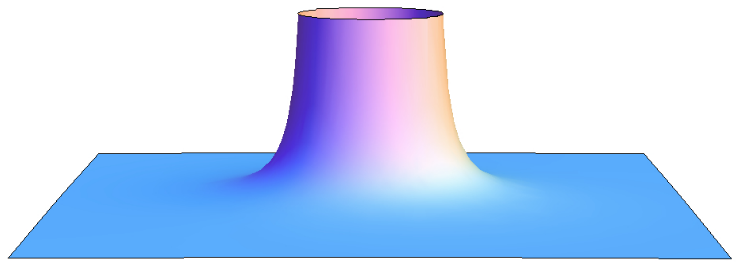
soln. in lin. regime: Maxwell point charge with delta-function source

➔ string ending in point charge on the brane



use full non-linear DBI: brane curves before it reaches the point charge

F-strings dissolving into D-brane



described by an embedding profile that follows from DBI EOM

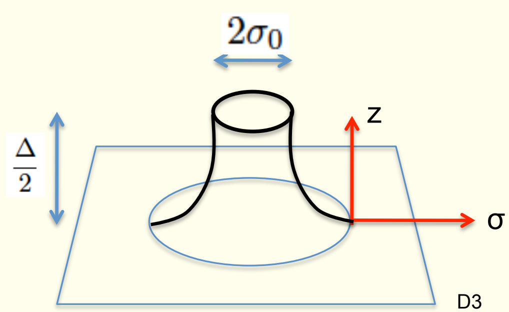
Setup and solution

specialize to: 10D flat background metric + zero 4-form

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + \sum_{i=1}^{6} dx_i^2$$

embedding of 3-brane $t = \tau\,,\ r = \sigma_1 \equiv \sigma\,,\ x_1 = z(\sigma)\,,\ \theta = \sigma_2\,,\ \phi = \sigma_3$

+ turn constant w.v. gauge field F_{01}.



- k F-strings ending on N coincident infinitely extended D3-branes
- or stretching between two parallel systems

$$z(\sigma) \to 0 \quad \text{for} \quad \sigma \to \infty$$

$$z'(\sigma) \to -\infty \quad \text{for} \quad \sigma \to \sigma_0$$

soln of EOM

$$z(\sigma) = \int_\sigma^\infty d\sigma' \frac{\sqrt{\sigma_0^4 + \kappa^2}}{\sqrt{\sigma'^4 - \sigma_0^4}}$$

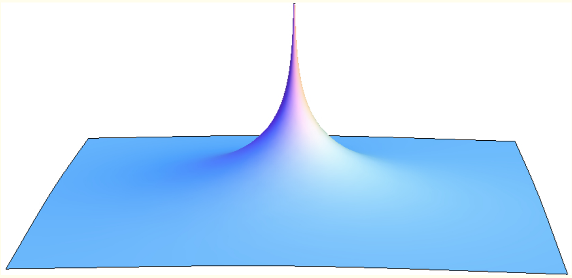
$$\kappa \equiv \frac{kT_{\rm F1}}{4\pi T_{\rm D3}} = k\pi g_s l_s^2.$$

Spike and wormhole solution (T=0)

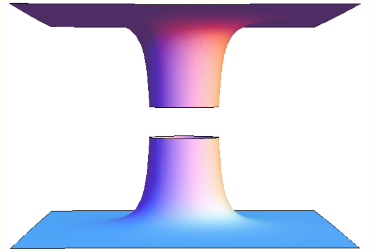
spike:

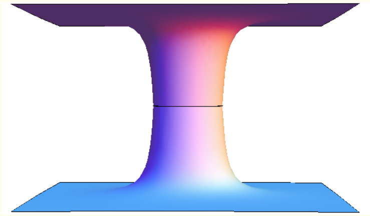
$$z(\sigma) = \frac{\kappa}{\sigma}$$

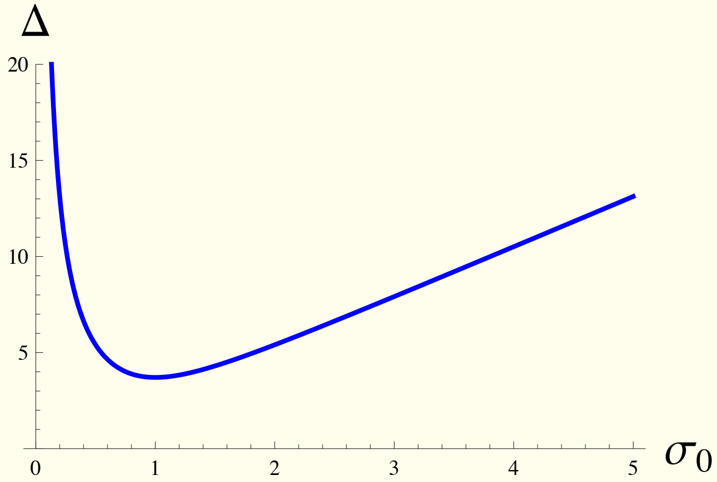
$$\left.\frac{dH}{dz}\right|_{\sigma=\sigma_0=0} = 4\pi T_{\mathrm{D3}}\kappa = kT_{\mathrm{F1}}$$



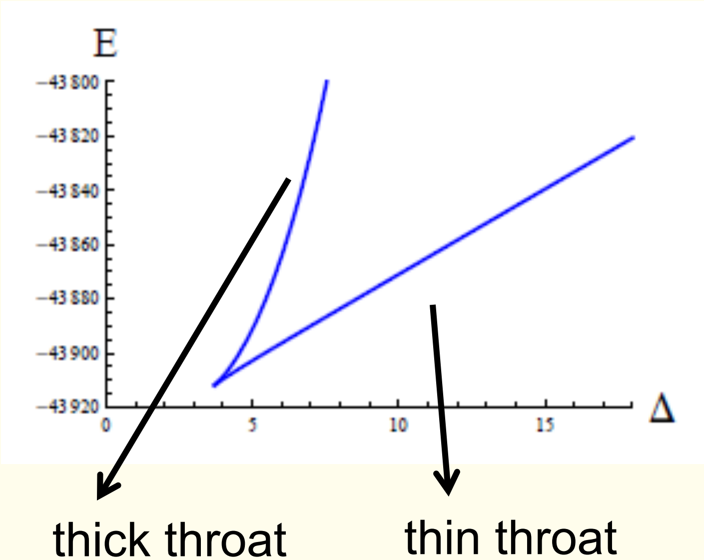
wormhole from solution with finite throat size:
attach mirror configuration







$$\Delta_{\min} \sim \sqrt{\kappa} \text{ at } \sigma_0 = \sqrt{\kappa}$$



Heating up the Bion using blackfold approach

Grignani,Harmark,Marini,NO,Orselli

go to regime in which we have larger number N of coincident D3-branes and large g_s N: compute EM tensor from sugra solution of D3-F1 bound st.

$$ds^2 = D^{-\frac{1}{2}} H^{-\frac{1}{2}} (-f dt^2 + dx_1^2) + D^{\frac{1}{2}} H^{-\frac{1}{2}} (dx_2^2 + dx_3^2) + D^{-\frac{1}{2}} H^{\frac{1}{2}} (f^{-1} dr^2 + r^2 d\Omega_5^2)$$

$$f = 1 - \frac{r_0^4}{r^4} \ , \quad H = 1 + \frac{r_0^4 \sinh^2 \alpha}{r^4} \ , \quad D^{-1} = \cos^2 \zeta + \sin^2 \zeta H^{-1}$$

+ other non-zero fields

solution depends on r0, alpha and zeta

read off stress tensor and D3-brane current

$$T^{00} = \frac{\pi^2}{2} T_{\rm D3}^2 r_0^4 (5 + 4 \sinh^2 \alpha) \ , \quad T^{11} = -\gamma^{11} \frac{\pi^2}{2} T_{\rm D3}^2 r_0^4 (1 + 4 \sinh^2 \alpha)$$

$$T^{22} = -\gamma^{22} \frac{\pi^2}{2} T_{\rm D3}^2 r_0^4 (1 + 4 \cos^2 \zeta \sinh^2 \alpha) \ , \quad T^{33} = -\gamma^{33} \frac{\pi^2}{2} T_{\rm D3}^2 r_0^4 (1 + 4 \cos^2 \zeta \sinh^2 \alpha)$$

$$J^{0123} = \frac{2\pi^2 T_{\rm D3}^2}{\sqrt{\gamma}} \cos \zeta r_0^4 \cosh \alpha \sinh \alpha$$

charged perfect fluid: r0, alpha, zeta depend on w.v. coordinate sigma
➔ dependence fixed by requiring constant T and N,k (charge conservation)

Action for thermal BIon

action for thermal Bion takes DBI-like form:

$$\mathcal{F}(T,N,k) = \frac{2T_{\mathrm{D3}}^2}{\pi T^4}\int_{\sigma_0}^{\infty} d\sigma \sqrt{1+z'(\sigma)^2}F(\sigma)$$

$$F(\sigma) = \sigma^2 \frac{1+4\sinh^2\alpha(\sigma)}{\cosh^4\alpha(\sigma)}$$

with alpha a function of w.v. coordinate sigma:

$$\cosh^2\alpha = \frac{3}{2}\frac{\cos\frac{\delta}{3}+\sqrt{3}\sin\frac{\delta}{3}}{\cos\delta} \qquad \cos\delta(\sigma) \equiv \bar{T}^4\sqrt{1+\frac{\kappa^2}{\sigma^4}}$$

definitions $\bar{T} \equiv \frac{T}{T_{\mathrm{bnd}}}$ $T_{\mathrm{bnd}}(N) \equiv \left(\frac{4\sqrt{3}T_{\mathrm{D3}}}{9\pi^2 N}\right)^{\frac{1}{4}}$ $\kappa \equiv \frac{kT_{\mathrm{F1}}}{4\pi N T_{\mathrm{D3}}}$

relation to DBI: $\lim_{T\to 0}\mathcal{F} = N H_{\mathrm{DBI}}$

Analytic solution

EOM can be integrated exactly

$$z'(\sigma) = -\left(\frac{F(\sigma)^2}{F(\sigma_0)^2} - 1\right)^{-\frac{1}{2}}$$

focus on branch connected to extremal (other one connected to neutral)

⇨ reproduces extremal BIon in zero temperature limit

$$-z'(\sigma) = \sqrt{\frac{\kappa^2 + \sigma_0^4}{\sigma^4 - \sigma_0^4}}\left[1 + \mathcal{O}(\bar{T}^4)\right]$$

$$\sim \frac{\kappa}{\sigma^2} \quad (\text{for } \sigma_0 = 0)$$

validity of the probe approximation:

$$r_{\text{c}}(\sigma) << \sigma$$

$$r_{\text{c}}(\sigma) << L_{\text{curv}} = |K^{-1}|$$

charge radius of the brane:

$$r_{\text{c}}^4 \sim \left(1 + \frac{\kappa^2}{\sigma^4}\right)\frac{N}{T_{\text{D3}}}$$

$$\sigma_0^3 \gg \sqrt{k} g_s l_s^3$$

Separation distance in finite T wormhole BIon

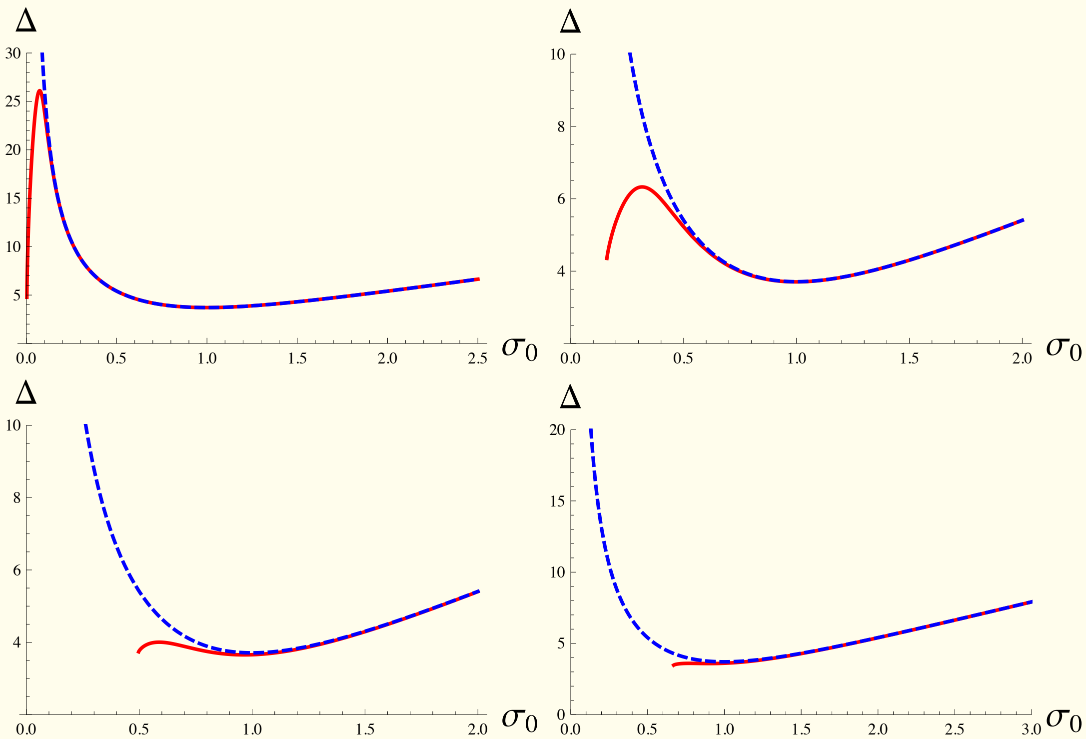


Figure 2: On the figures the solid red line is Δ versus σ_0 for $\bar{T} = 0.05$ (top left figure), $\bar{T} = 0.4$ (top right figure), $\bar{T} = 0.7$ (bottom left figure) and $\bar{T} = 0.8$ (bottom right figure) while the blue dashed line corresponds to $\bar{T} = 0$. We have set $\kappa = 1$.

new feature: three (or one phases) for given Delta instead of two (at zero T)

-brane separation cannot become arbitrarily large on thin throat branch

- for low T large part of curve like zero T, but still max brane separation
- large T: only thick throat branch

Comparison of phases in canocial ensemble

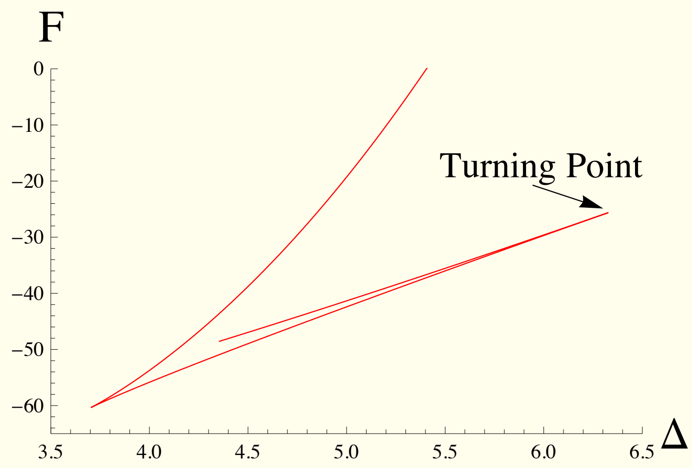


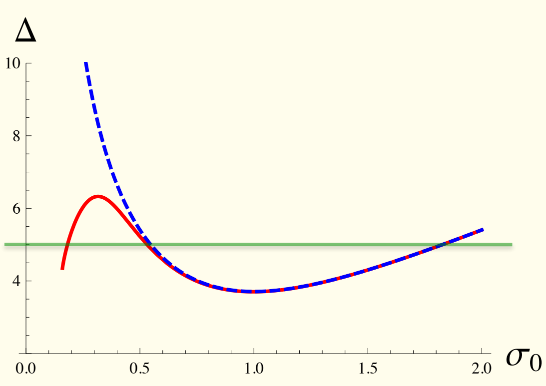
Figure 4: The free energy $\delta\mathcal{F}$ versus Δ for $\bar{T} = 0.4$ and $\kappa = 1$.

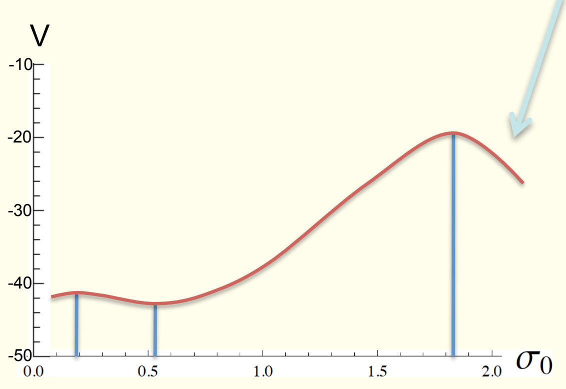
which of phases dominate ?

- for Delta below max value: the thin throat branch has lowest free energy
- for Delta above max value: unstable saddle point

(critical temperature)

heuristic picture





perturbing leads to time-dep soln in which wormhole throat increases and brane anti-brane system disappers (tachyon condensation ?)

Thermal spike ?

finite T analogue of Bion does not allow for infinite spike

configuration in question can be made by matching up a non-extremal black F-string solution with throat solution

non-trivial: two independent gluing conditions, tension & entropy density agree impressively good