Thermodynamics of the brane in Chern-Simons matter theories with flavor

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Based on work with N. Jokela, J. Mas & A. V. Ramallo
1211.6045 & work in progress

Summary

Review of the ABJM model

Addition of backreacted massless flavor

Flavored ABJM Black Hole

Brane Thermodynamics

ABJM Chern-Simons-matter theories

0806.1218

- describes the dynamics of multiple M2-branes at a $\mathbb{C}^4/\mathbb{Z}_k$ singularity

- Chern-Simons matter theory in 2+1 dim with gauge group $U(N)_k \times U(N)_{-k}$

- Field content (bosonic): 2 gauge fields & 2 pairs of chiral superfields, transforming in the $(N, \bar{N})$ bifundamental representations

- $\mathcal{N} = 6$ SUSY & $\lambda \sim \frac{N}{k}$ → rank of the gauge group / CS level

- When N & k are large $\overset{\text{type IIA}}{\Rightarrow}$ $AdS_4 \times \mathbb{CP}^3$ + fluxes with 24 SUSYs

$$ds^2 = L^2\, ds^2_{AdS_4} + 4\, L^2\, ds^2_{\mathbb{CP}^3}$$

$$L^4 = 2\pi^2 \frac{N}{k}$$

Flavor in Chern-Simons-matter systems in 2+1

Hohenegger & Kirch 0903.1730

Massless quarks $\Rightarrow$ D6-branes extending in AdS_4 & wrapping $\mathbb{RP}^3 \subset \mathbb{CP}^3$

Introduce quarks in the $(N,1)$ and $(1,N)$ representation

$$Q_1 \to (N,1) \qquad Q_2 \to (1,N) \qquad \tilde{Q}_1 \to (\bar{N},1) \qquad \tilde{Q}_2 \to (1,\bar{N})$$

coupling to vector multiplet & bifundamentals

Introducing backreaction

$$S_{WZ} = T_{D_6} \sum_{i=1}^{N_f} \int_{\mathcal{M}_7^{(i)}} \hat{C}_7 \to T_{D_6} \int_{\mathcal{M}_{10}} C_7 \wedge \Omega$$

Modified BI

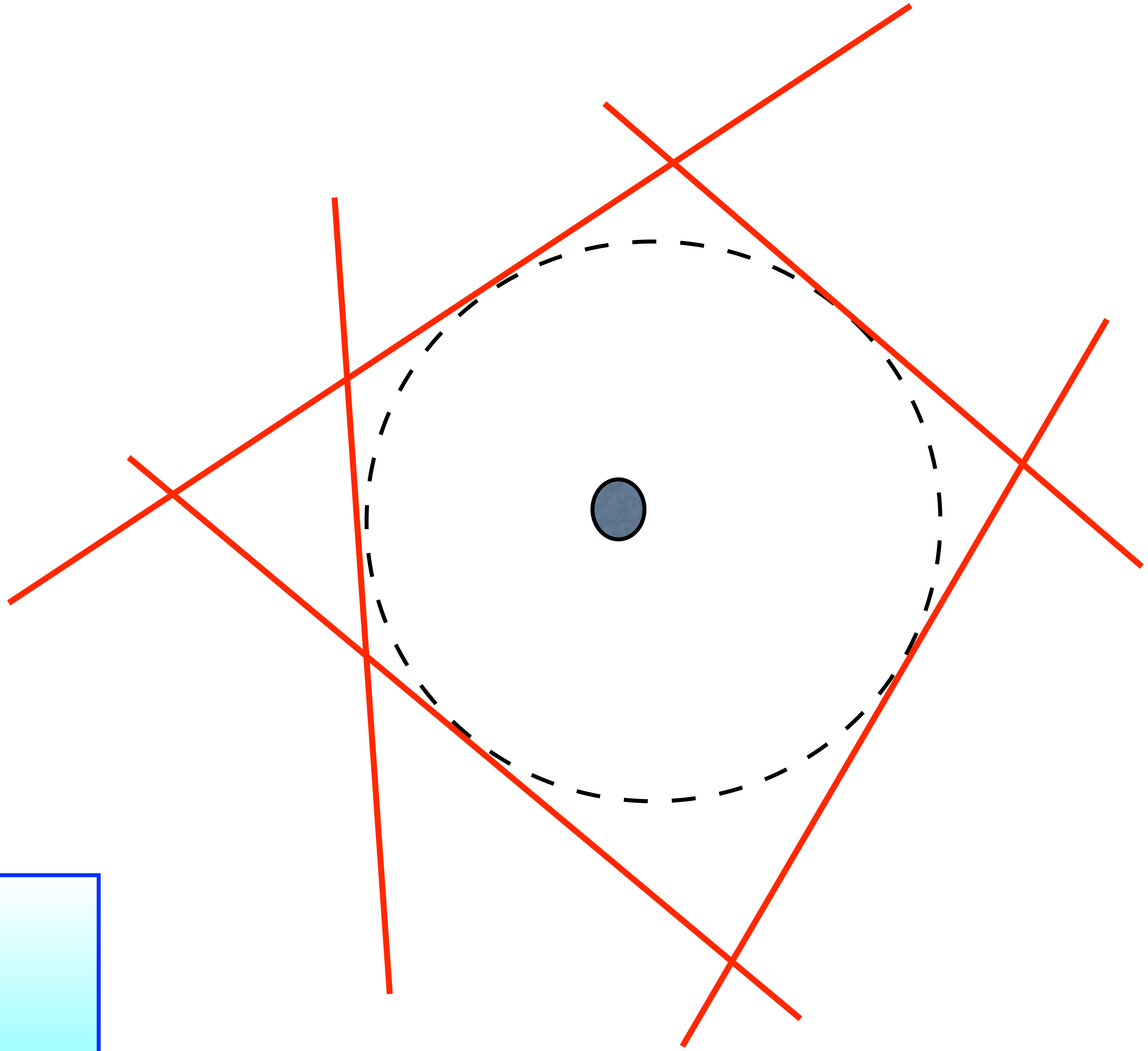
$$dF_2 = 2\pi\, \Omega$$

$U(N_f)$

Localized & coincident massless flavors $\Rightarrow$ $AdS_4 \times \mathcal{M}_7$

Conformality with flavor

Smeared mass(less) sources



- -no delta-function sources
- -still can preserve (less) SUSY
- -much simpler (analytic) solutions
- $-$flavor symmetry : $U(1)^{N_f}$

Unquenching ABJM with smeared flavor

Conde & Ramallo 1105.6045

Write the $\mathbb{CP}^3$ & the RR 2-form as

$$ds^2_{\mathbb{CP}^3} = \frac{1}{4}\left[ds^2_{\mathbb{S}^4} + \left(dx^i + \epsilon^{ijk} A^j x^k\right)^2\right]$$

$\mathbb{S}^2$-bundle over $\mathbb{S}^4$ with $A^i \to SU(2)$ instanton on $\mathbb{S}^4$

$\mathcal{S}^i \to$ (rotated) basis of one-forms along $\mathbb{S}^4$

$E^i \to$ one-forms along the $\mathbb{S}^2$ fiber

$$F_2 = \frac{k}{2}\left(E^1 \wedge E^2 - (\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2)\right)$$

Squash the metric & the RR 2-form as

$$ds^2 = L^2\, ds^2_{AdS_4} + ds^2_6$$

$$ds^2_6 = \frac{L^2}{b^2}\left[q\, ds^2_{\mathbb{S}^4} + \left(dx^i + \epsilon^{ijk} A^j x^k\right)^2\right]$$

$q \to \mathbb{CP}^3$ internal squashing

$b \to$ relative $AdS_4/\mathbb{CP}^3$ squashing

Deformation parameter

$$\eta \equiv 1 + \frac{3N_f}{4k}$$

$$F_2 = \frac{k}{2}\left[E^1 \wedge E^2 - \eta\,(\mathcal{S}^4 \wedge \mathcal{S}^3 + \mathcal{S}^1 \wedge \mathcal{S}^2)\right]$$

Dilaton & RR 4-form

$$e^{-\phi} = \frac{b}{4}\,\frac{\eta+q}{2-q}\,\frac{k}{L}$$

$$F_4 = \frac{3kb}{4}\,\frac{\eta+q}{2-q}\,L^2\,\Omega_{AdS_4}$$

Regime of validity $\qquad L >> 1\,, \qquad e^{\phi} << 1$

$$\frac{N_f}{k} \sim 1 \quad \Rightarrow \quad N^{\frac{1}{5}} << k << N$$

(same as in the unflavored case)

$$N_f >> k \quad \Rightarrow \quad N^{\frac{1}{5}} << N_f << N$$

Flavored ABJM black hole

$$ds^2 = L^2 \, ds^2_{BH_4} + ds^2_6$$

$$ds^2_{BH_4} = -r^2 h(r) dt^2 + \frac{dr^2}{r^2 h(r)} + r^2 \left[(dx^1)^2 + (dx^2)^2 \right] \qquad h(r) = 1 - \frac{r_h^3}{r^3}$$

Temperature

$$T = \frac{3\, r_h}{4\pi}$$

Free energy & entropy

$$F_{back} = -\frac{1}{9} \left(\frac{4\pi}{3} \right)^2 \frac{N^2}{\sqrt{2\lambda}} \, \xi \left(\frac{N_f}{k} \right) T^3$$

$$s_{back} = \frac{1}{3} \left(\frac{4\pi}{3} \right)^2 \frac{N^2}{\sqrt{2\lambda}} \, \xi \left(\frac{N_f}{k} \right) T^2$$

reflection of the conformality

Massive flavor to the Flavored black hole

D6 extends in x^μ, r & $\mathbb{RP}^3$
- -2 directions inside $\mathbb{S}^4$
- -1 direction inside $\mathbb{S}^2$

↓ Write the $\mathbb{S}^2$ metric as $ds^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

D6 extends in φ with a profile in $\theta(r)$

↓ Introduce new Cartesian-like coordinates

$$\rho = r^b \sin\theta$$

$$R = r^b \cos\theta$$

Total action $= DBI + WZ$ ➡ $S_{WZ} = T_{D6} \int \hat{C}_7$

$\mathcal{C}_7 = e^{-\phi}\mathcal{K}$ has to be improved to get consistent thermo

$$\delta C_7 = \frac{L^7 q}{b^3}\, e^{-\phi}\, d^3x \wedge \Big[L_1(\theta)\, d\theta \,+\, L_2(r)\, dr \Big] \wedge \Xi_3 \qquad d(\delta C_7) = 0$$

Requiring the angular part of $\hat{\mathcal{C}}_7$ to vanish at the horizon

$$L_1(\theta) = -\frac{r_h^3}{b} \sin\theta \cos\theta$$

Jensen 1006.3066

Total action

$$S = -\mathcal{N}_r \int d^3x\, dr\, r^2 \sin\theta \Big[\sqrt{1 + \frac{r^2\, h(r)}{b^2}\, \dot{\theta}^2} - \sin\theta - \frac{r h(r)}{b} \cos\theta\, \dot{\theta} \Big] + \mathcal{N}_r\, r_h^3 \int d^3x\, \Delta_0$$

$$\int dr\, L_2(r) \equiv r_h^3\, \Delta_0$$

- Δ_0 does not depend on the embedding $\Rightarrow$ zero point energy
- $\mathcal{N}_r(\frac{N_f}{k}) \Rightarrow$ flavor determines the size of the cycle

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\Big|_{r=r_h} = 0$$ ⇒ no momentum flow through the horizon ⇒ Horizon is not a dynamical surface

Free energy density ⇒ $$F = -\frac{S}{\int d^3x}$$

Decoupling of quarks with infinite mass

$$\lim_{m\to\infty} F = \left(\frac{1}{4b} - \Delta_0\right) r_h^3 \mathcal{N}_r = 0 \quad \longrightarrow \quad \boxed{\Delta_0 = \frac{1}{4b}}$$

Non-trivial test → massless flavors → background and probe flavors are of the same type

↓ expand the flavor function $\xi(N_f)$ as

$$\xi\left(\frac{N_f+1}{k}\right) = \xi\left(\frac{N_f}{k}\right) + \xi'\left(\frac{N_f}{k}\right)\frac{1}{k} + \cdots$$

←

$$\Delta F_{back} = -\left(\frac{4\pi}{3}\right)^2 \frac{N^2}{9\sqrt{2\lambda}} \frac{1}{k} \xi'\left(\frac{N_f}{k}\right) T^3$$

↓ equating

$$\Delta F_{back} = F(m=0) \quad \rightarrow \quad \xi' = \frac{3}{4b}\zeta$$

highly non-trivial match!!!

Entropy: as the probe falls through the horizon it increases the area by 1 unit

$$s_{total} \overset{m\to 0}{\approx} \frac{1}{3}\left(\frac{4\pi}{3}\right)^2 \frac{N^2}{\sqrt{2\lambda}} \xi\left(\frac{N_f+1}{k}\right) T^2$$

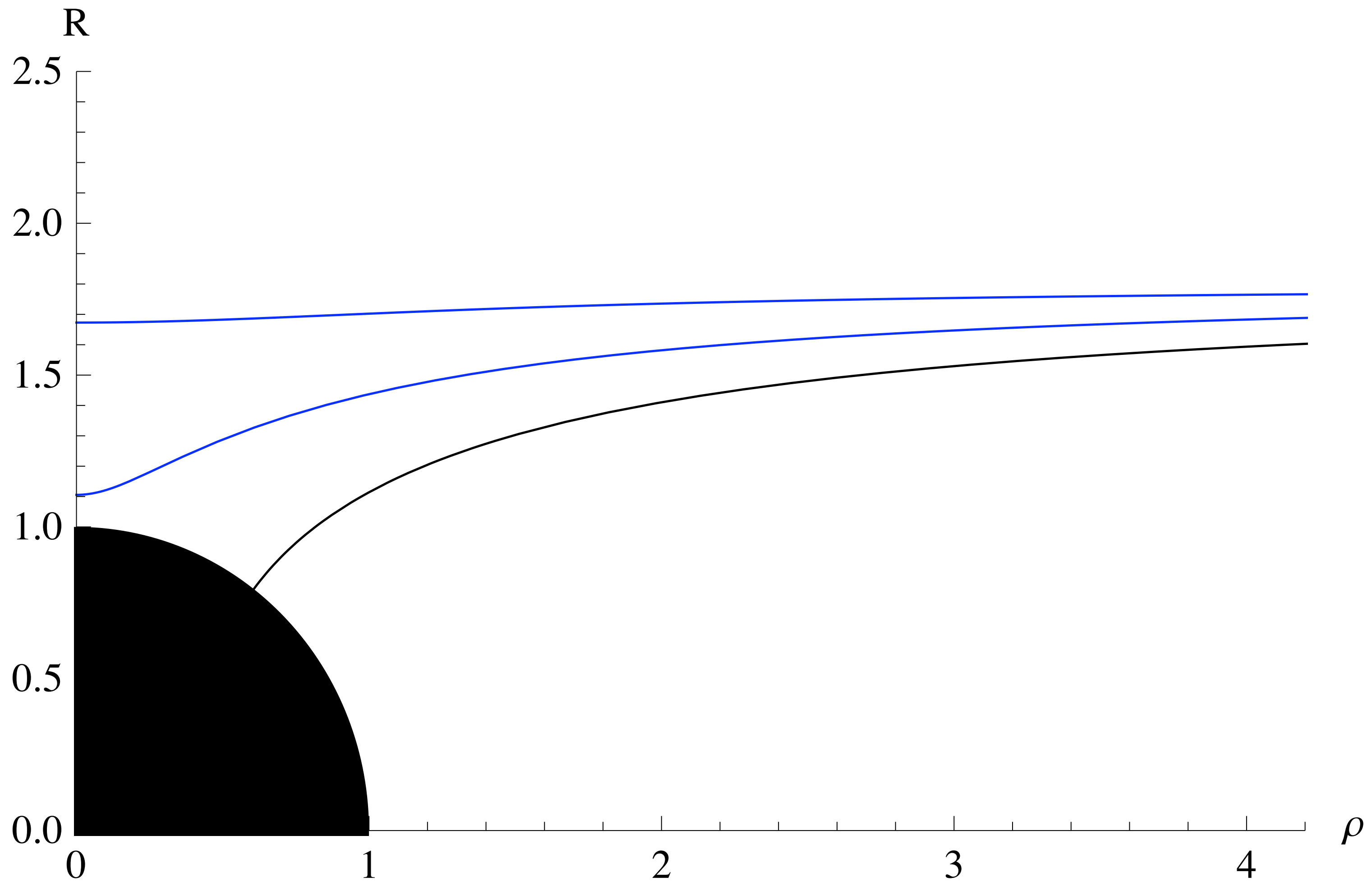
Different embeddings

Minkowski: smoothly close above the horizon

Black hole: end at the horizon

Critical: between MN & BH

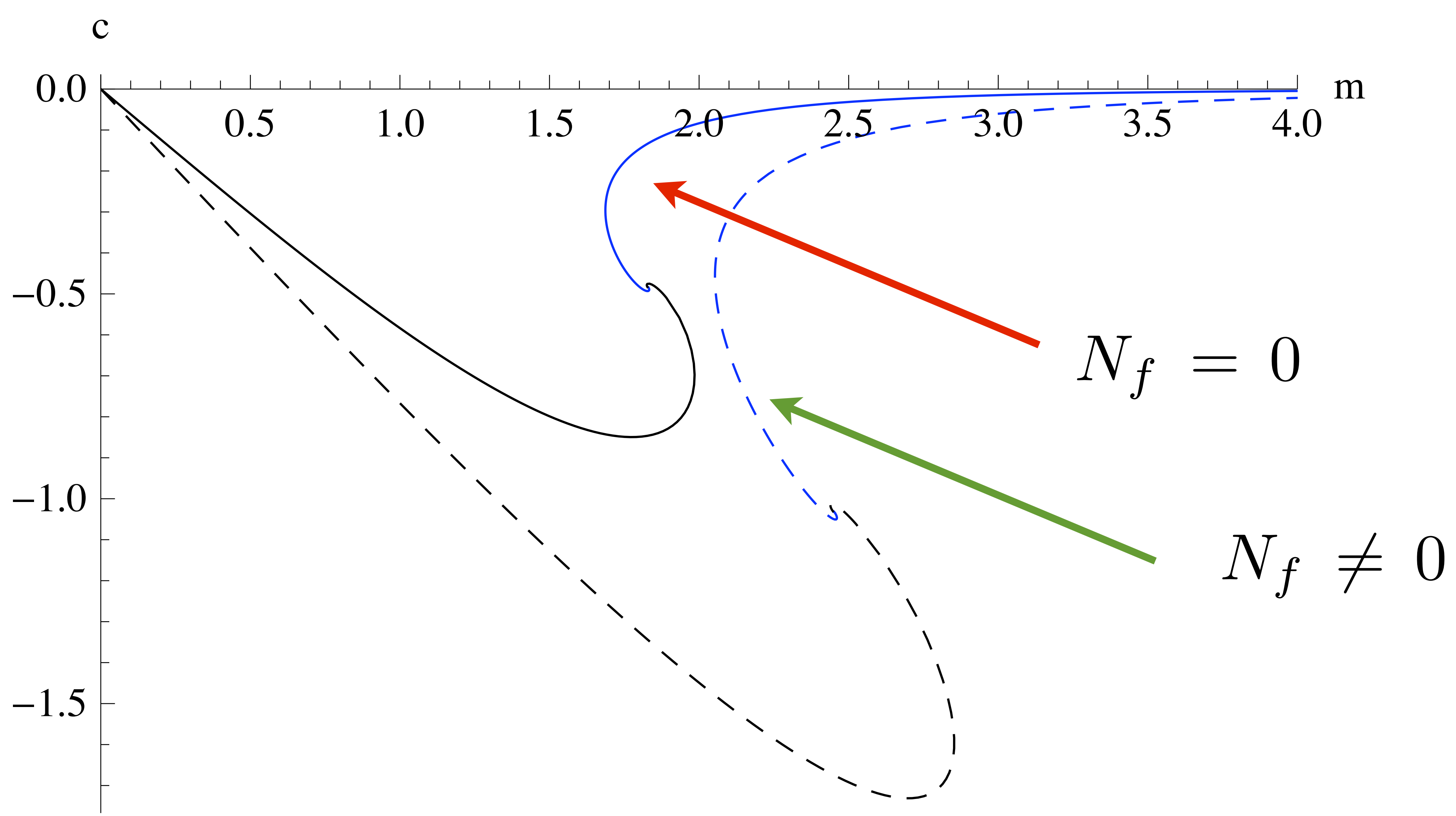
Asymptotically → $R \sim m + \frac{c}{\rho^{\frac{3}{b}-2}}$



Holographic Dictionary

$$m_q = \frac{2^{\frac{1}{3}}\pi}{3}\,\sqrt{2\lambda}\,\sigma T m^{\frac{1}{b}}$$

$$\langle \mathcal{O}_m \rangle = -\frac{2^{\frac{2}{3}}\,\pi}{9}\,\frac{(3-2b)\,b}{q}\,\sigma\,N\,T^2\,c$$



Free energy

$$\frac{F}{\mathcal{N}} = \mathcal{G}(m) - 1$$

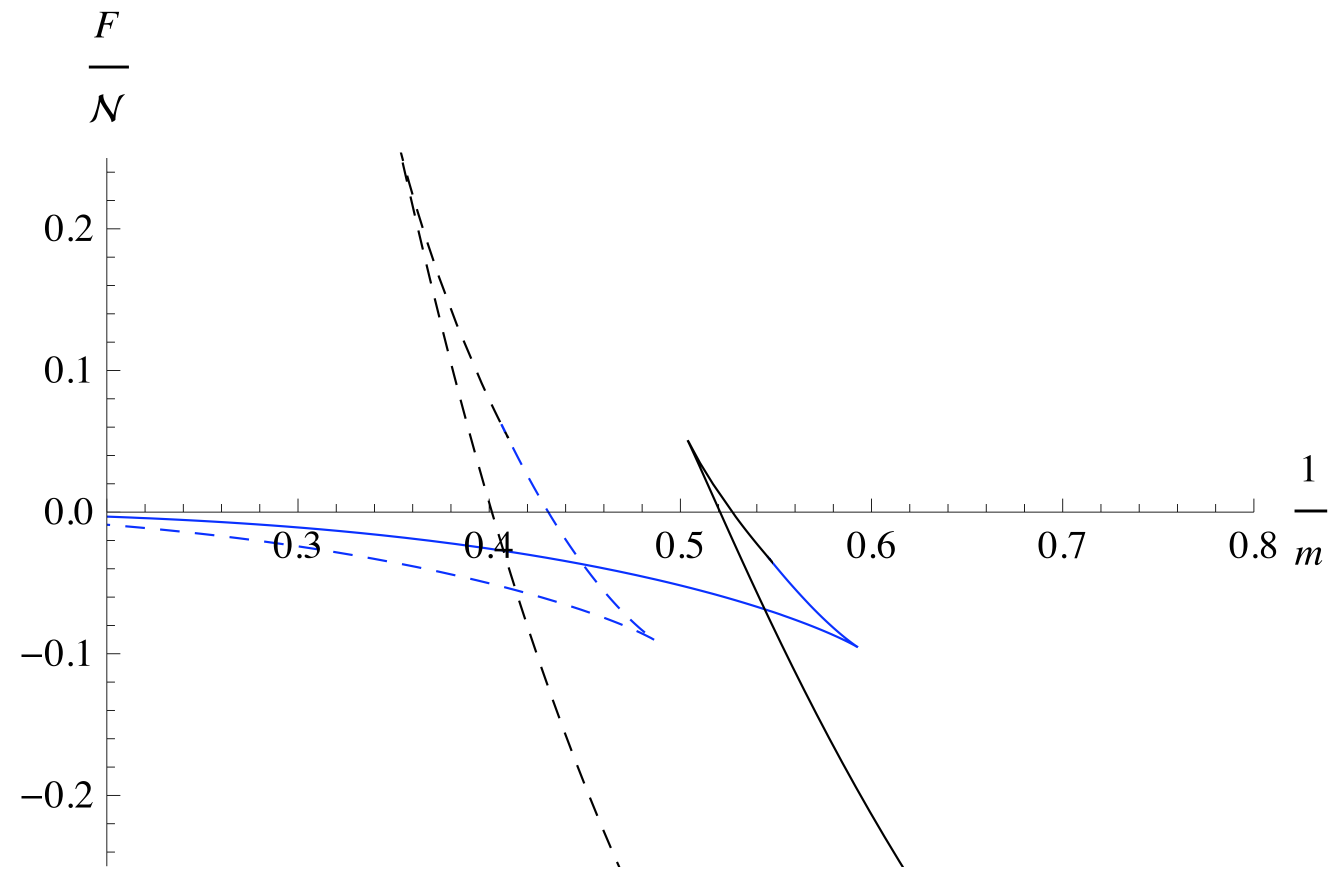
an integral of the action over the embedding

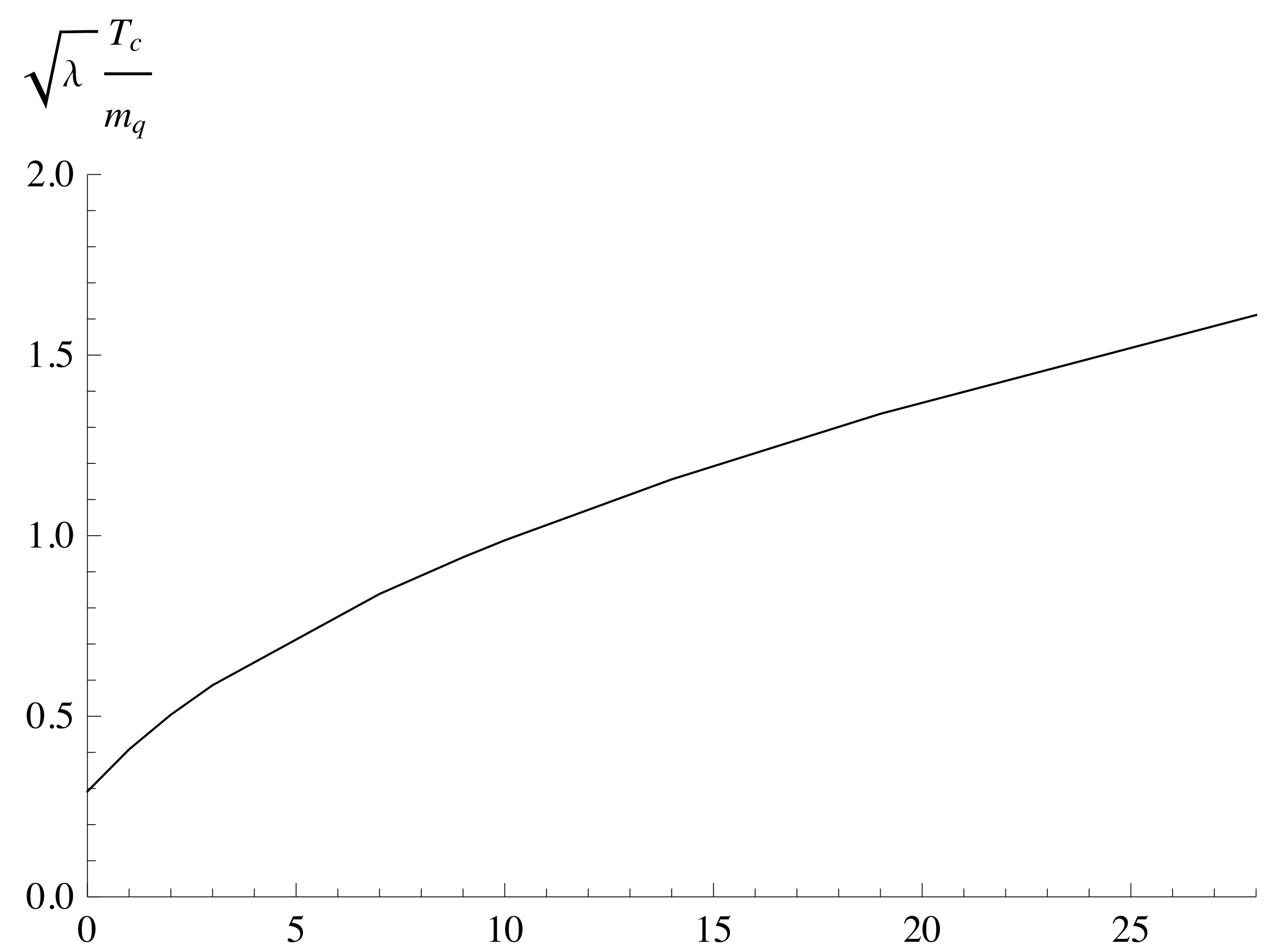
$$\frac{F}{\mathcal{N}} \approx -\frac{2b}{3+2b}\left(\frac{T}{\bar{M}}\right)^3$$

$$\frac{F}{\mathcal{N}} \approx -1 + \frac{3}{b}\left[\frac{\Gamma\left(1-\frac{b}{3}\right)}{\Gamma\left(\frac{1}{2}-\frac{b}{3}\right)}\right]^2 \tan\left(\frac{\pi b}{3}\right)\left(\frac{\bar{M}}{T}\right)^{2b}$$

First order PT between a gapped & a gapless phase. Critical temperature increases with flavor

$$\sqrt{\lambda}\,\frac{T_c}{m_q} \overset{N_f >> 1}{\propto} \sqrt{N_f}$$



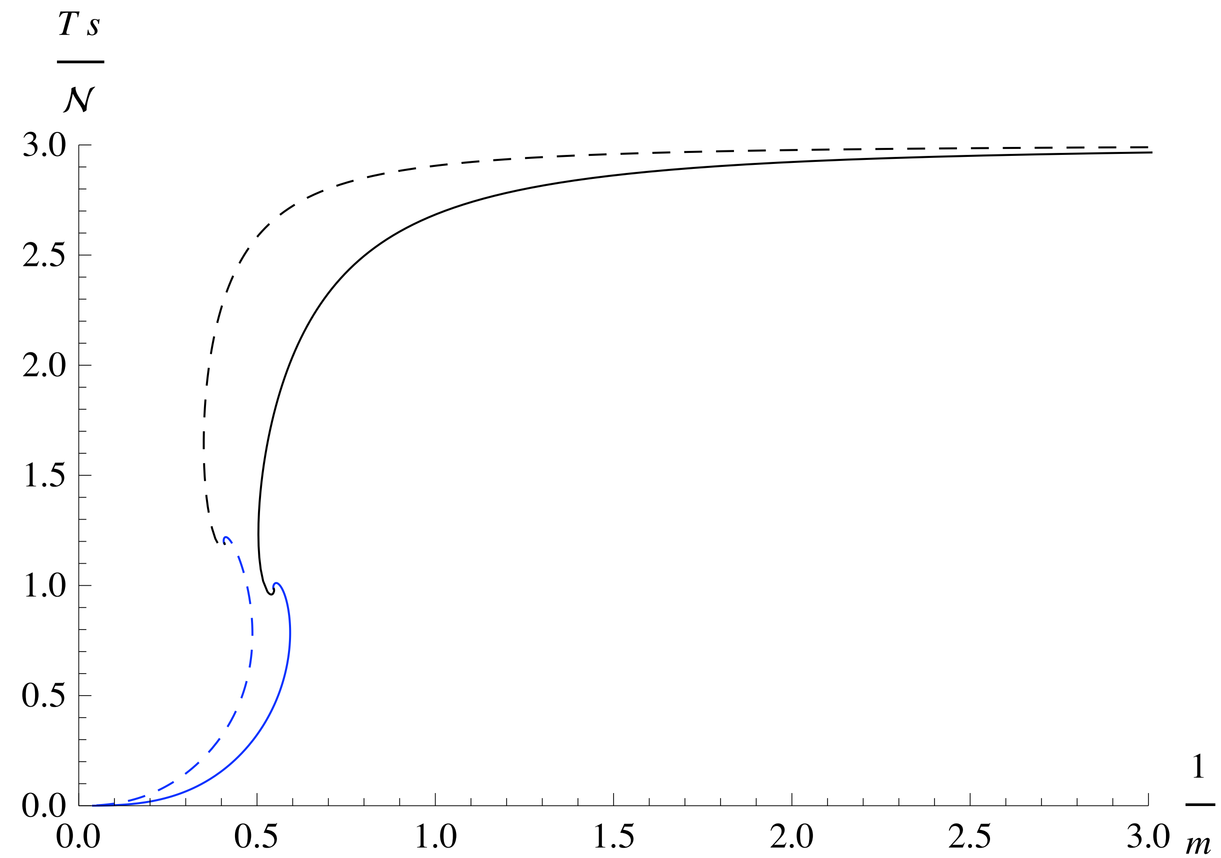


Entropy

$$T\,\frac{s}{\mathcal{N}} = -3\,\mathcal{G}(m) + 3 - (3-2b)\,c\,m$$

$$T\,\frac{s}{\mathcal{N}} \approx \frac{12b}{2b+3}\left(\frac{T}{\bar{M}}\right)^3$$

$$T\,\frac{s}{\mathcal{N}} \approx 3 - \frac{3(3-2b)}{b}\left[\frac{\Gamma\left(1-\frac{b}{3}\right)}{\Gamma\left(\frac{1}{2}-\frac{b}{3}\right)}\right]^2 \tan\left(\frac{\pi b}{3}\right)\left(\frac{\bar{M}}{T}\right)^{2b}.$$

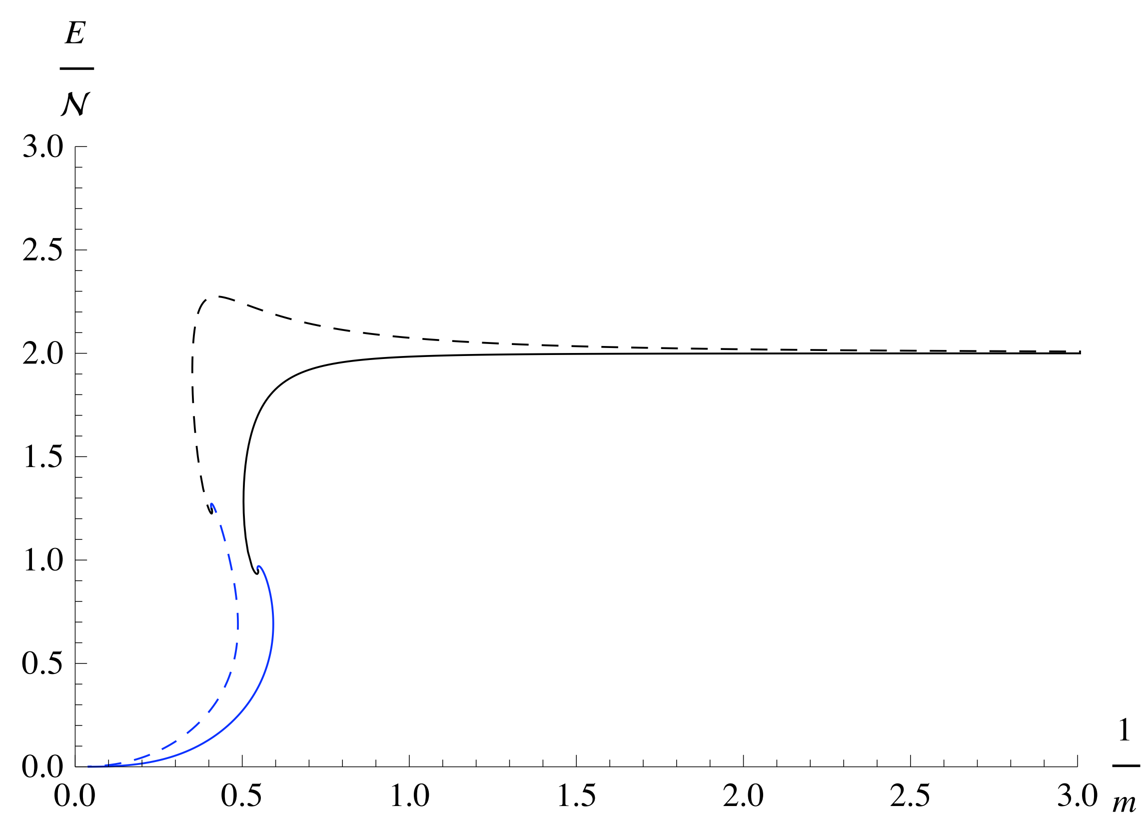


Internal energy

$$\frac{E}{\mathcal{N}} = -2\,\mathcal{G}(m) + 2 - (3-2b)\,c\,m$$

$$\frac{E}{\mathcal{N}} \approx \frac{10b}{2b+3}\left(\frac{T}{\bar{M}}\right)^3$$

$$\frac{E}{\mathcal{N}} \approx 2 + \frac{6(b-1)}{b}\left[\frac{\Gamma\left(1-\frac{b}{3}\right)}{\Gamma\left(\frac{1}{2}-\frac{b}{3}\right)}\right]^2 \tan\left(\frac{\pi b}{3}\right)\left(\frac{\bar{M}}{T}\right)^{2b}$$

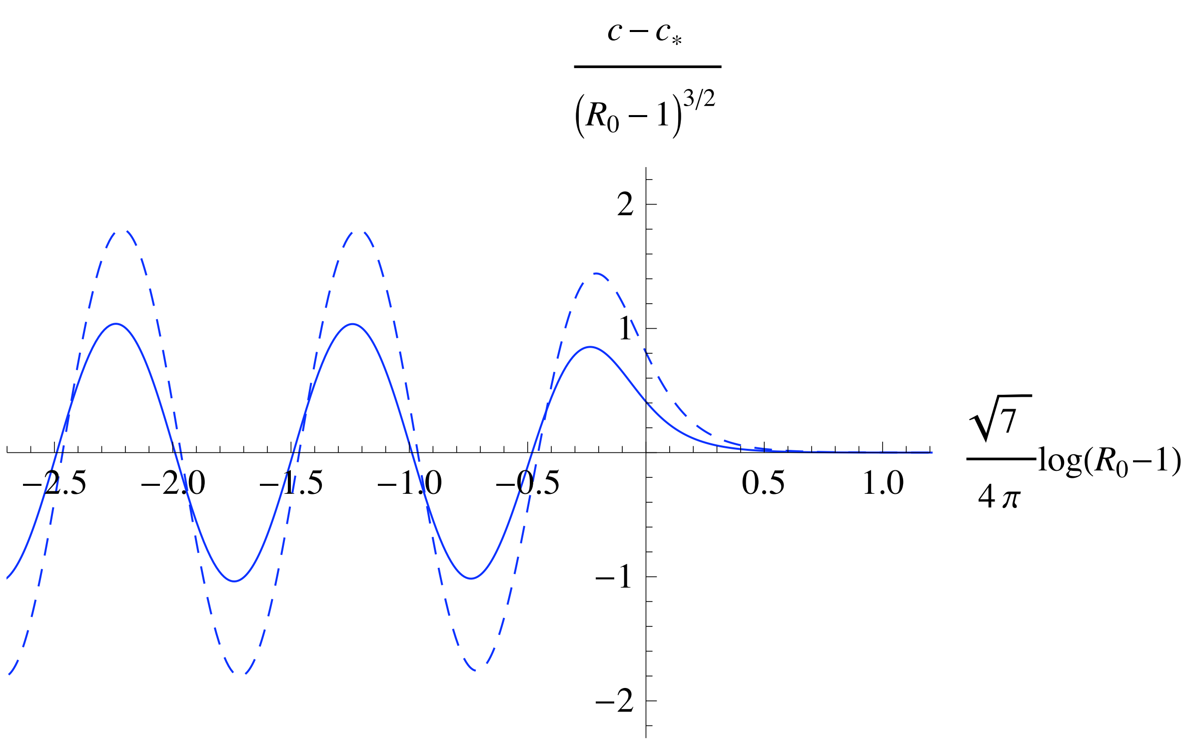


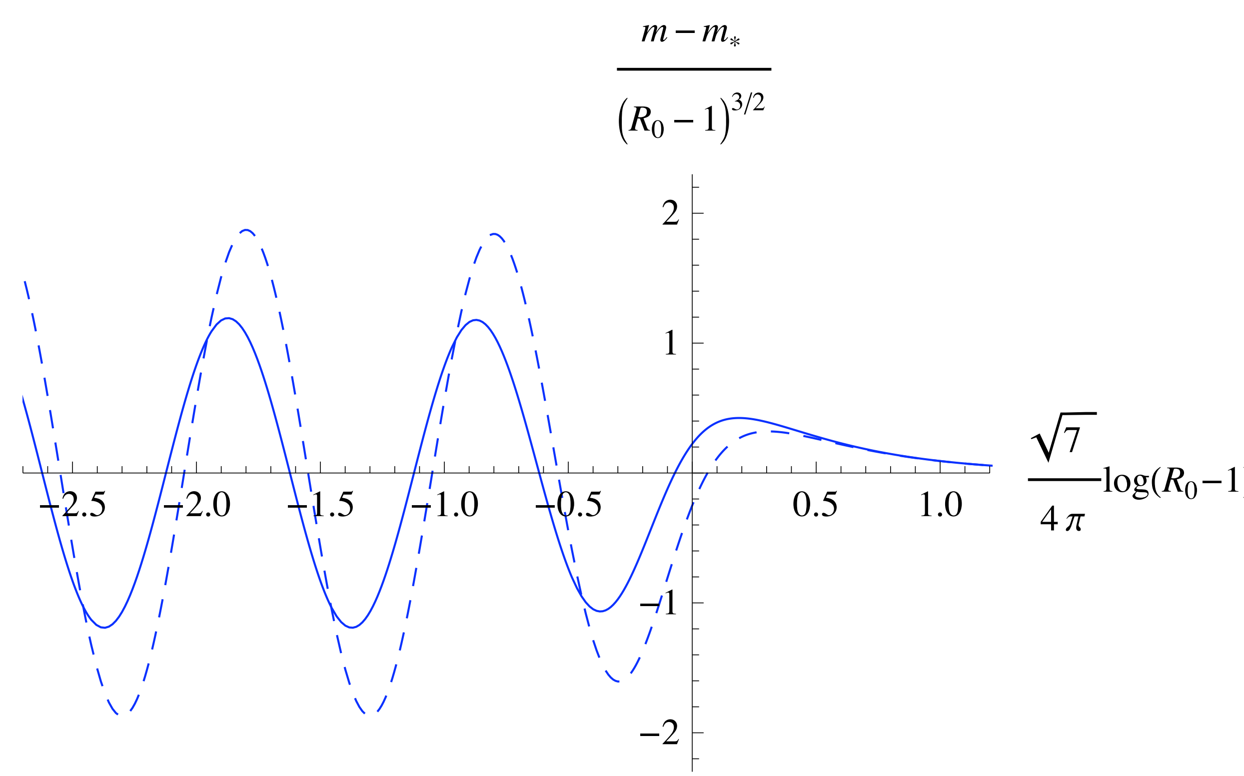
No numerical derivatives to compute those quantities!!!

Near the transition - critical embeddings

Zooming in the spiral with a logarithmic microscope we see a self similar behavior

c & m oscillate





Future directions

- Meson spectrum → meson melting transition

- Addition of more gauge fields

- charge density & magnetic field → Condensed Matter Physics