Model-independent Resonance Parameters

Saša Ceci, Alfred Švarc, Branimir Zauner

Ruđer Bošković Institute

with special thanks to

Mike Sadler, Shon Watson, Jugoslav Stahov, Pekko Metsa, Mark Manley and Simon Capstick

Helsinki 2007

The 4th International PWA Workshop

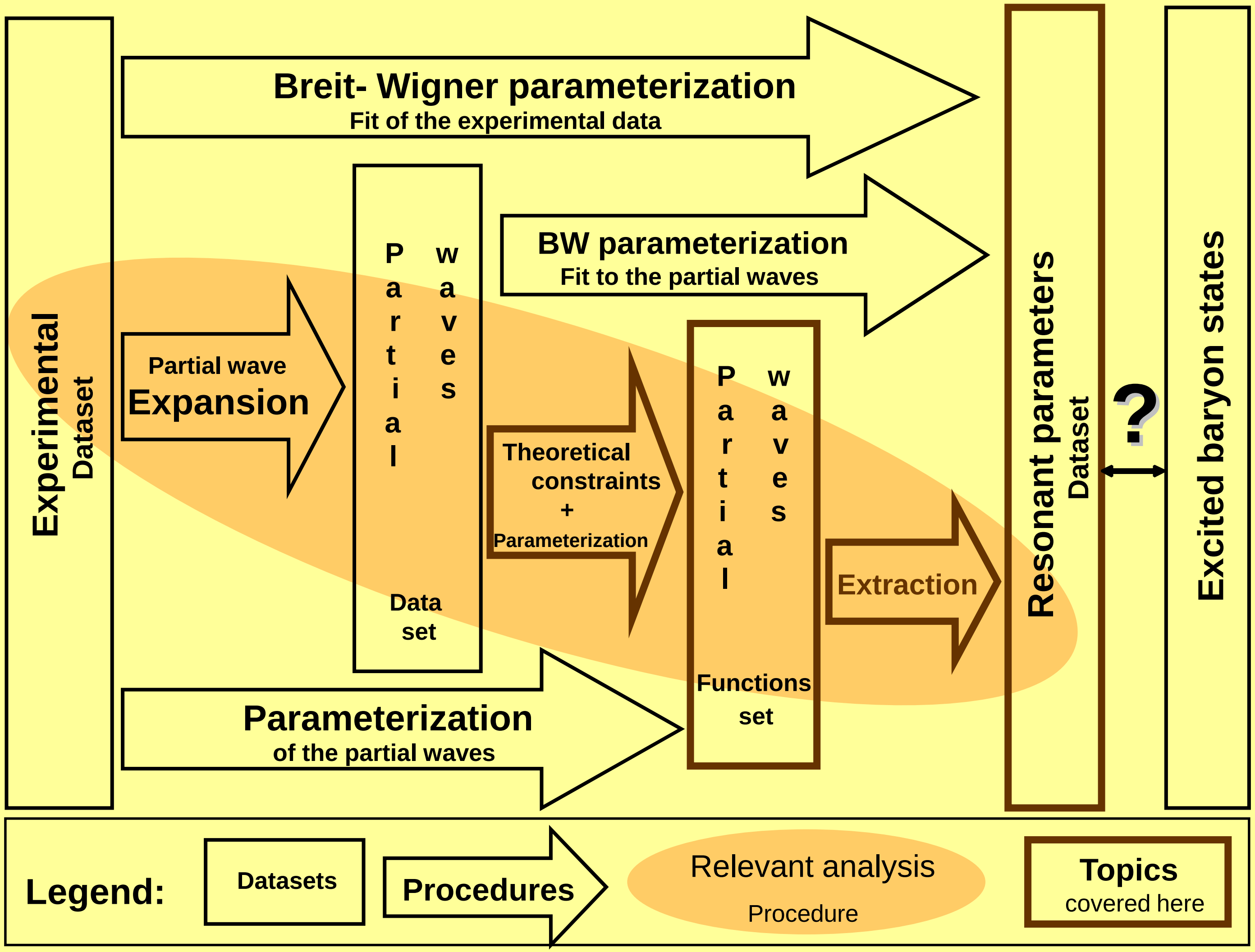
Abstract

model-independent extraction
of the
resonance parameters
is enabled by imposing of the
physical constraints
to the
scattering matrix.

Resonant Scattering

- strong peaks of the cross section of the meson-nucleon scattering: a manifestation of the resonance phenomena,
- resonances – excited baryons,
- baryon spectroscopy – obtaining of the resonance parameters (masses and widths) from the scattering data,
- at energies close to the baryon-resonance masses, the microscopic model (QCD) is still insolvable.

How would one obtain resonance parameters



The Relevant Analyses

What are relevant analyses

- Arndt FA02
- Manley MS92
- Hoehler KH80
- Cutkosky CMB (‘79)

Origin: *Review of Particle Physics (PDG).*

Getting spectra can be difficult

- partial-wave functions are obtained from the numerical calculations (DR, expansions on nontrivial functions),
- there are too many models, and it is not clear which one (and in which cases) should be used,
- in order for many extraction methods to work properly, some “additional” requirements must be met.

It was (allegedly) easier in the old days ...

- **Transition matrix - T**
  - $T = T_R + T_B$
  - like adding of the Feynman diagrams

or

- **Caley's transform of unitary scattering matrix - K**
  - $K = K_R + K_B$
  - the scattering matrix unitarity is conserved

or

- **Phase shift - δ**
  - $\delta = \delta_R + \delta_B$
  - addition of potentials

... when people were using these simple resonant contributions ...

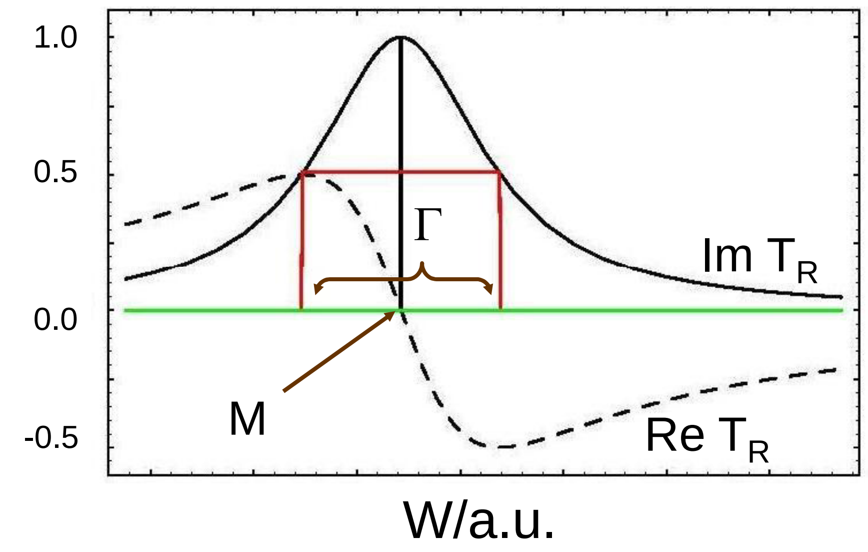
- **Resonant matrix T_R**
  - $T_R = (\Gamma/2) / (M - W - i\,\Gamma/2)$

or

- **Resonant matrix K_R**
  - $K_R = (\Gamma/2) / (M - W)$

or

- **Resonant phase shift - δ_R**
  - $\delta_R = \arctan[(\Gamma/2) / (M - W)]$



... but, was it really?

- **T-matrix addition**
  - generally violates the S-matrix unitarity,
- **K-matrix addition**
  - conserves unitarity, but the approach is not unique,
- **Phase shift addition**
  - comes down to S-matrix multiplication (yet another model),
  - in multichannel cases, order of matrix multiplications is not defined (instead of the scalar δ, we have matrix Δ, coming from $S = e^{2i\Delta}$).

What are, then, resonant parameters?

Natural attempts

- Matrices T and K have poles:
  - $T = K\,(I - i\,K)^{-1}$,
  - $K = T\,(I + i\,T)^{-1}$.
- S has common poles with T:
  - $S = I + 2\,i\,T$.
- What could be done with δ?

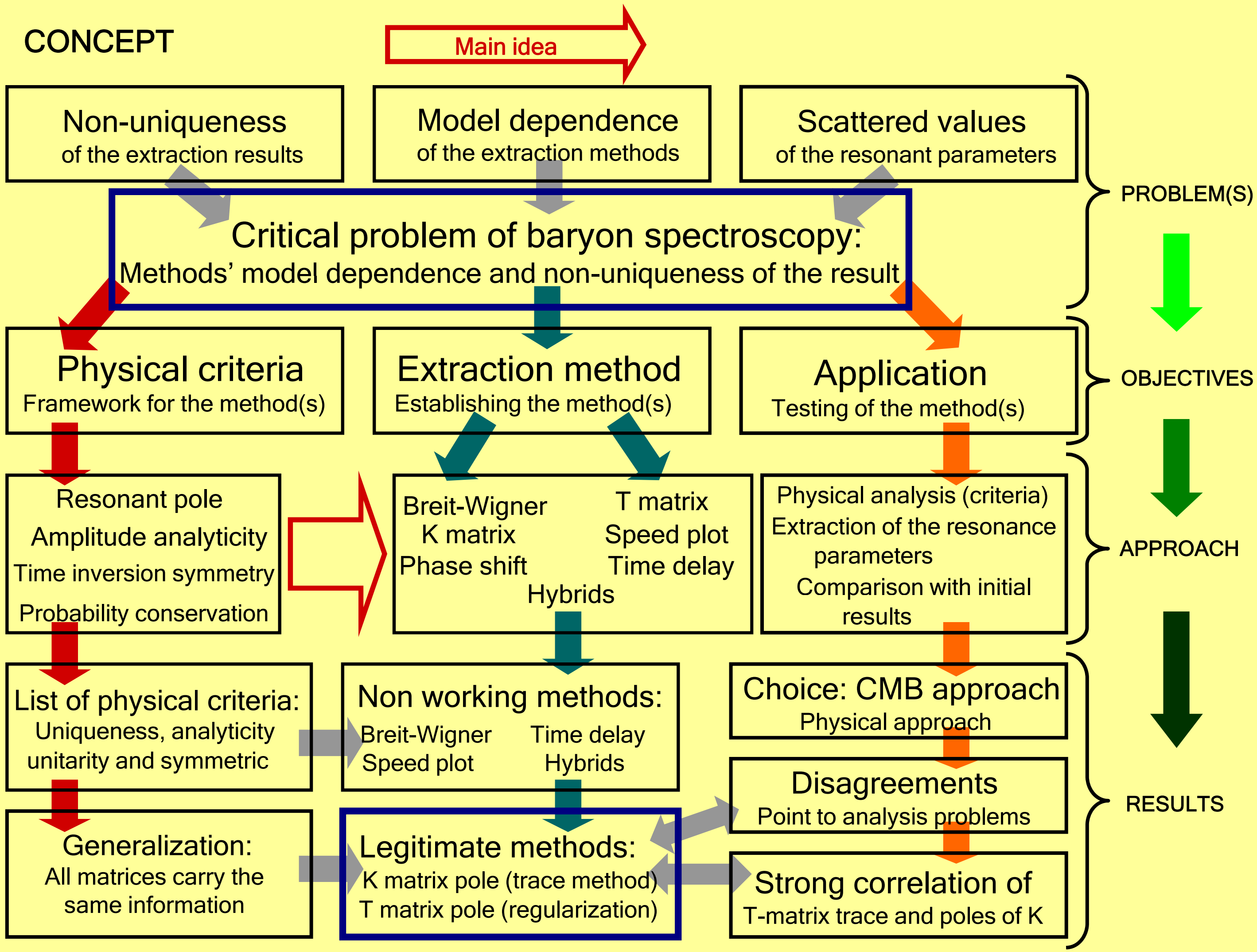
In baryon spectroscopy

- Pole parameters:
  - *pole of T matrix*
- Breit-Wigner parameters:
  - *fit of the BW parameterization to the T (or K?) matrix*

The Choices We Must Make: picking the resonance contribution

- there are many possible choices – **<u>how to pick the right one</u>**?
- it is much simpler if the scattering is:
  - single channel (all inelastic channels closed),
  - single resonant (just one resonance contributes),
  - with constant resonant "parameters".
- **generalization** to **multichannel** and **multiresonance** situations, with <u>nontrivial energy dependence</u> of **background** contributions and **resonant parameters** calls for quite **elaborate approaches**,
- **<u>The Assumption</u>:** energy dependent partial waves has been established properly,
- <u>Now we must **determine** proper resonance parameters, and **extract**</u> them.

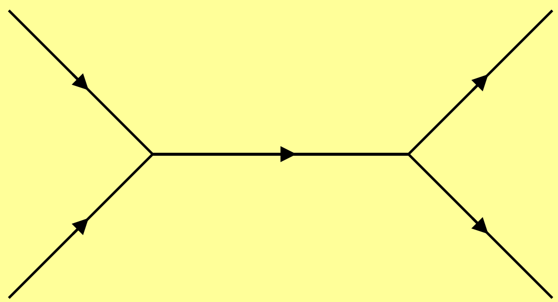
- ***To find physical criteria,***
- ***To determine resonant parameters based on those criteria,***
- ***To test (i.e. apply) method on well known example.***



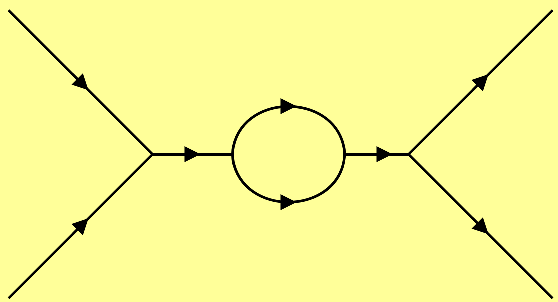
Physical criteria

Physical conditions		Scattering matrix
• **probabilities** add up to one,	⇐	• scattering matrix **unitarity**,
• **time-inversion invariance,**	⇐	• **symmetrical** scattering matrix,
• **parity** is conserved in QCD,	⇒	• **parity** is a good quantum number,
• **total spin** is also conserved,	⇒	• **spin** is a good quantum number,
• quantum and relativistic mechanics – occurrence of the **propagators**	⇒	• **poles** emerges.

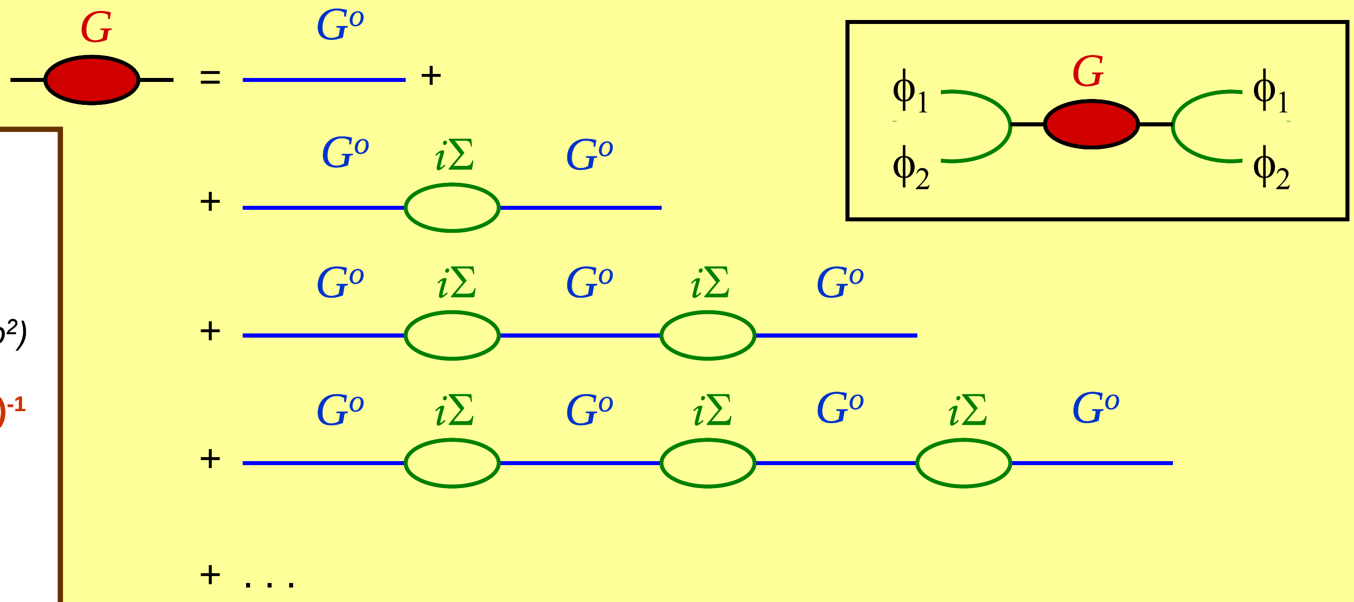
Infinite cross section?!



- Propagator G
- $G^o = i\,(p^2 - m^2 + i\varepsilon)^{-1}$
- $G^{(1)} = G^o$
- masses of products greater then the propagator mass – infinite contribution to cross section at some physical energy (simple pole)



- next order – loop contribution $i\Sigma$
- $G^{(2)} = G^o + G^o\, i\Sigma\, G^o$
- even worse – pole is now second order



$G^o = i\,(p^2 - m^2 + i\varepsilon)^{-1}$

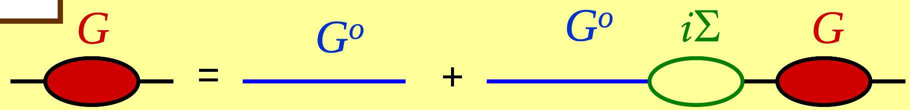
$m^2 \rightarrow m^2 - i\Sigma$

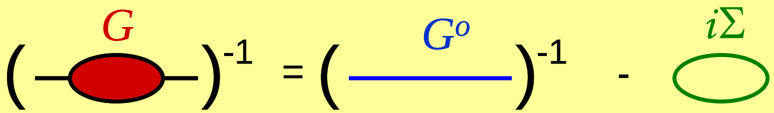
$i\Sigma = i\,p\Gamma f + \mathrm{Re}$, $p = sqrt(p^2)$

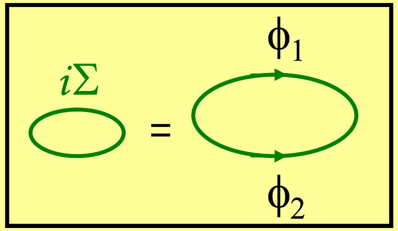
$G = i\,(p^2 - m^2 + ip\Gamma f + \mathrm{Re})^{-1}$

Standard Breit-Wigner:
Re = 0, f = 1, p = m.

Flatte approximation:
Re = 0, f = 1.







Scattering matrix **unitarity**

- scattering matrix is unitary so it can be diagonalized by some unitary matrix U
  - $S = U^{+} S_D U$
- S_D is diagonal matrix
  - $S_D = \sum s_a E^a$
  - matrix E^a is a vector of orthonormal basis for diagonal matrix decomposition
- We define matrices χ^a
  - $\chi^a = U^{+} E^a U$
- S matrix expansion
  - $S = \sum \chi^a s_a = \sum \chi^a e^{2 i \delta_a}$

Matrix χ properties

•	hermiticity	$(\chi^a)^{+} = \chi^a$
•	orthoidempotence	$\chi^a \chi^b = \chi^a \delta^{ab}$
•	completeness	$\sum \chi^a = I$
•	trace	$\mathrm{Tr}\, \chi^a = 1$

$$E^1 = \begin{pmatrix} 1 & 0 & \cdot & 0 \\ 0 & 0 & \cdot & \cdot \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdot & \cdot & 0 \end{pmatrix},\ E^2 = \begin{pmatrix} 0 & 0 & \cdot & 0 \\ 0 & 1 & \cdot & \cdot \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdot & \cdot & 0 \end{pmatrix},\ \cdots,\ E^N = \begin{pmatrix} 0 & \cdot & \cdot & 0 \\ \vdots & \ddots & \vdots & \vdots \\ \cdot & \cdot & 0 & 0 \\ 0 & \cdot & 0 & 1 \end{pmatrix}$$

The rest of the physical demands

- Other physical conditions on matrix χ:
  - $\chi^I{}_J\pi$, $\delta^I{}_J\pi$ — **parity, spin** and **isospin** conservation
  - $\chi^T = \chi$ — **time inversion** invariance
  - $\chi^a = O^T E^a O$ — χ matrix has no **poles** on the real axis (orthogonal O instead of unitary U).
- **Poles** come from δ_a and other functions of δ_a:
  - $\sin \delta_a \, e^{i\delta_a}$, $\tan \delta_a$, $e^{2i\delta_a}$
- A new question:
  - What is going on with χ outside of the real axes?

<u>**Generalization**</u> of S-matrix "offshoots" by χ matrices

S-matrix "offshoots"

- matrices T, K i Δ may be derived from S matrix,
- matrices S, T, K i Δ carry **the same information** – what differs them is **our capability** to extract it,
- what is interesting is the fact that matrices K, Im T i Re S are diagonalized by the same (real) orthogonal matrices (O).

$$S = \sum_{a=1}^{N} s_a \chi^a \quad : \quad s_a = e^{2i\delta_a},$$

$$\Delta = \sum_{a=1}^{N} \delta_a \chi^a \quad : \quad \delta_a = \delta_a,$$

$$T = \sum_{a=1}^{N} t_a \chi^a \quad : \quad t_a = e^{i\delta_a} \sin \delta_a,$$

$$K = \sum_{a=1}^{N} k_a \chi^a \quad : \quad k_a = \mathrm{tg}\, \delta_a,$$

Trace – channel-dependence elimination

- $T = \Sigma \chi^a t_a$ / Tr   ⇐ $\mathrm{Tr}\, \chi^a = 1$

Tr T =	$\Sigma\, t_a$	=	$\Sigma \sin \delta_a\, e^{i\delta_a}$	=	$t_r + t_b$
Tr K =	$\Sigma\, k_a$	=	$\Sigma\, \mathrm{tg}\, \delta_a$	=	$k_r + k_b$
Tr Δ =	$\Sigma\, \delta_a$	=	$\Sigma\, \delta_a$	=	$\delta_r + \delta_b$
Tr S =	$\Sigma\, s_a$	=	$\Sigma\, e^{2i\delta_a}$	=	$s_r + s_b$

Trace of the matrix χ is 1 on the real axis:

- is it χ (i.e. Tr χ) analytic function?
- if it is, it will have value 1 in the vicinity of the real axis as well (in the resonant area);
- then we can say this: *Pole positions of the T matrix, as well as those of the K matrix, will depend on the energy dependence of phase shift δ, and will not care about the energy dependence of χ.*

Resonant parameter **extraction**: types of parameters

S or T have pole	(pole T)
K has pole	(pole K)
Δ has "something" $\pi/2$	(?)
δ_a is $\pi/2$	(pole K)
$\tan(\delta_a)$ has pole	(pole K)
speed plot shows peak	("pole T")
time delay shows peak	("pole T")
imaginary part of T matrix peak	(BW?)

Basically, two different ways:

- T-matrix pole
- K-matrix pole

But there could be some other possibilities ...

Up-to-date resonant parameters **definitions**

T-matrix pole

$T_R = r / (\mu - W)$

Speed plot

$|dT_R/dW| = |r| / [(\mathrm{Re}\ \mu - W)^2 + (\mathrm{Im}\ \mu)^2]$

Time delay

$d\delta_R/dW = (\Gamma/2) / [(M - W)^2 + \Gamma^2/4]$

Hybrids

?

K-matrix pole

$K_R = (\Gamma/2) / (M - W)$

Breit-Wigner fit

$T_R = (\Gamma/2) / (M - W - i\Gamma/2)$

Phase shift is $\pi/2$

$\delta_R = \arctan[(\Gamma/2) / (M - W)]$

Poles of K and T matrices

$$k_r = \frac{\Gamma_r(W)/2}{M_r(W) - W}$$

$$t_r = \frac{\Gamma_r(W)/2}{M_r(W) - W - i\,\Gamma_r(W)/2}$$

These are not (necessarily) parameterizations!

$$\begin{aligned}
S &= \chi^r \frac{M_r - W + i\,\Gamma_r/2}{M_r - W - i\,\Gamma_r/2} + \sum_{a \neq r}^{N} \chi^a s_a, \\
\Delta &= \chi^r \,\mathrm{arctg}\left(\frac{\Gamma_r/2}{M_r - W}\right) + \sum_{a \neq r}^{N} \chi^a \delta_a, \\
K &= \chi^r \frac{\Gamma_r/2}{M_r - W} + \sum_{a \neq r}^{N} \chi^a k_a, \\
T &= \chi^r \frac{\Gamma_r/2}{M_r - W - i\,\Gamma_r/2} + \sum_{a \neq r}^{N} \chi^a t_a.
\end{aligned}$$

K-matrix Poles: The Recipe I

$$K = T/(I + iT)$$

(i) The parameter extraction procedure starts when a full T matrix has been obtained from an energy-dependent partial-wave analysis of experimental data.

(ii) Contrary to the usual prescription, where Eq. (11) is used to obtain resonance parameters from the T matrix in a model-dependent way, we use Eq. (7) to obtain the full K matrix from the known T matrix.

(iii) Poles of Tr K are found to obtain a set of resonance masses $M_1^R, \cdots M_{N_R}^R$, where N_R is the number of resonances.

(iv) Multiplying both sides of Eq. (14) by $(M_k^R - W)$ and setting the energy W to the value of the kth resonance mass (M_k^R), the corresponding resonance width is isolated:

$$\Gamma_k^R = 2 \lim_{W \to M_k^R} \left[\left(M_k^R - W \right) \mathrm{Tr}(K) \right]. \quad (16)$$

All other contributions to the K matrix trace, i.e. background, other resonances, and channel-couplings, are removed in this limiting process (this relation turns out to be similar to Eq.(16) in Ref. [10] for the case of the various πN isospin channels).

(v) The branching ratio of a resonance to a given channel can be obtained in similar manner, but this time using the diagonal K-matrix element, K_{aa} from Eq. (10) and definition (12)

$$x_a^k = \frac{2}{\Gamma_k^R} \lim_{W \to M_k^R} \left[\left(M_k^R - W \right) K_{aa} \right], \quad (17)$$

where, as before, all undesired contributions vanish.

(vi) Steps (iv) and (v) are then repeated for all resonances found in (iii).

S. Ceci, A. Švarc, B. Zauner, D. M. Manley and S. Capstick, ***hep-ph/0611094v1***

T-matrix Poles:
Regularization Method

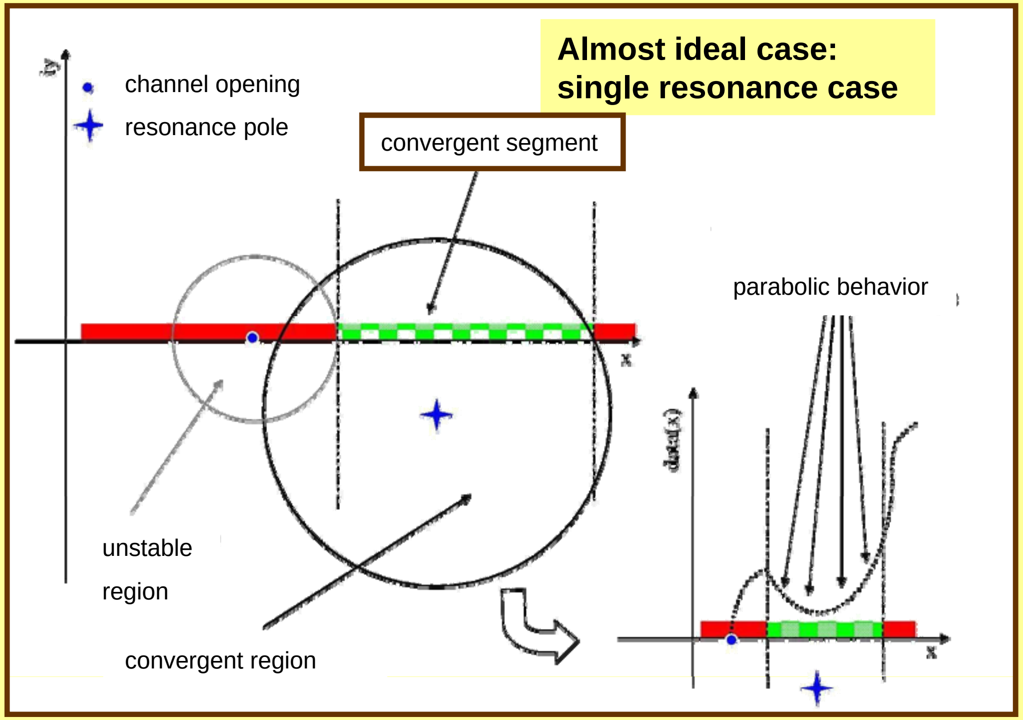
$$f(z) = (\mu - z)\, t(z)$$

$$f(\mu) = \sum_{n=0}^{N} \frac{f^{(n)}(x)}{n!}(\mu - x)^n + R_N(x, \mu).$$

$$f^{(n)}(x) = (\mu - x)\, t^{(n)}(x) - n\; t^{(n-1)}(x),$$

$$|f(\mu)| = \frac{\left|t^{(N)}(x)\right|}{N!}\, |a + i\, b - x|^{(N+1)}.$$

$$\frac{(a-x)^2 + b^2}{\sqrt[N+1]{|f(\mu)|^2}} = \sqrt[N+1]{\frac{(N!)^2}{|t^{(N)}(x)|^2}}.$$



T-matrix Poles: The Recipe II

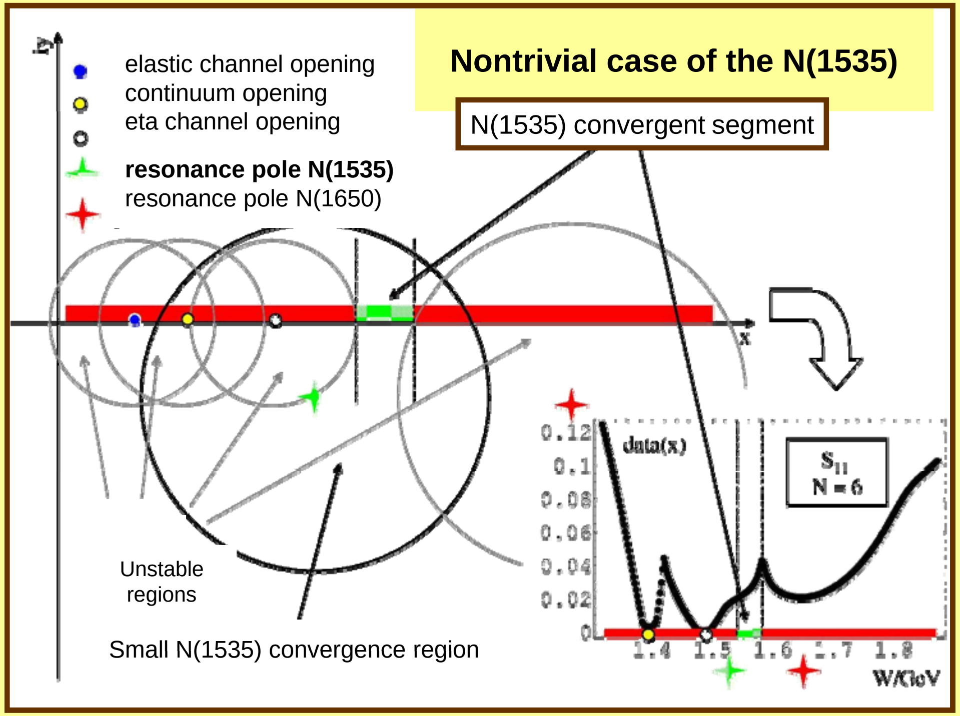
The dataset was produced by the right-hand side of Eq. (13). $T^{(N)}(x)$ was given by numerical derivation of energy-dependent partial waves obtained in our analysis [6]. A step of 2 MeV provided a stable procedure. The Taylor expansion is considered to converge when the extracted parameters settle down. The higher orders were used to obtain more accurate values of pole parameters. To acquire reliable fit results, we considered data grouped in a parabolic shape (in accordance with the second-order polynomial).

The elastic-pole residue is commonly given [5] by its absolute value $|r|$ and its phase θ, namely

$$|r| = |f(\mu)|, \quad \tan\theta = \mathrm{Im}\, f(\mu)\, /\, \mathrm{Re}\, f(\mu), \qquad (14)$$

where this particular (elastic) $f(\mu)$ is given by the first term in Eq. (11) with $T(z)$ being πN elastic T-matrix element.

Pole parameters attained in this way from πN elastic process are given in Table I. In order to verify the procedure, we applied it to other channel processes. Contrary to anomalous results obtained when using standard procedures, inelastic poles varied by only a few MeV from the elastic ones.



S. Ceci, J. Stahov, A. Švarc, S. Watson and B. Zauner, ***hep-ph/060923v1***

What is **in fact** speed plot?

$$\frac{(a-x)^2+b^2}{\sqrt[N+1]{|f(\mu)|^2}} = \sqrt[N+1]{\frac{(N!)^2}{|t^{(N)}(x)|^2}}.$$

***N* = 1**

$$\frac{(a-x)^2+b^2}{|f(\mu)|} = \frac{1}{|dt(x)/dx|},$$

$$\left|\frac{dt(W)}{dW}\right| = \frac{|f(\mu)|}{(a-W)^2+b^2}.$$

Speed plot

$|dT_R/dW| = |r| / [(\mathrm{Re}\ \mu - W)^2 + (\mathrm{Im}\ \mu)^2]$

OK, but who does really care what the speed plot in fact is?

- KH80 pole parameters are obtained by using of the speed plot (the relevant values),
- it is generally considered as a nifty way of finding pole position (NSTAR 2004),
- many PWA groups use it (NSTAR 2007),
- Excited Baryon Analysis Center (EBAC) at JLab in fact wants to study the validity of the speed plot and time delay (this years whitepaper),
- it is considered as a model-independent extraction method,
- many papers utilizing speed plot or time delay are already published, and some might be published as we speak.

Up-to-date resonant parameters **definitions**

T-matrix pole

$T_R = r / (\mu - W)$

Speed plot

$|dT_R/dW| = |r| / [(\mathrm{Re}\,\mu - W)^2 + (\mathrm{Im}\,\mu)^2]$

T-matrix pole: first order approximation

Time delay

$d\delta_R/dW = (\Gamma/2) / [(M - W)^2 + \Gamma^2/4]$

T-matrix pole: first order approximation

Hybrids

?

Model dependent

K-matrix pole

$K_R = (\Gamma/2) / (M - W)$

Breit-Wigner fit

$T_R = (\Gamma/2) / (M - W - i\Gamma/2)$

Model dependent

Phase shift is $\pi/2$

$\delta_R = \arctan[(\Gamma/2) / (M - W)]$

K-matrix pole

Which came first ?

<u>K-matrix</u> and its poles ⇄ **<u>T-matrix</u> and its poles**

Choice of the analysis

CMB

- unitary (S)
- analytic
- multiresonance
- coupled channel

$$\underbrace{\begin{pmatrix}\boxtimes & \boxtimes & \boxtimes\\ \boxtimes & \boxtimes & \boxtimes\\ \boxtimes & \boxtimes & \boxtimes\end{pmatrix}}_{T} = \underbrace{\begin{pmatrix}\boxplus & 0 & 0\\ 0 & \boxplus & 0\\ 0 & 0 & \boxplus\end{pmatrix}}_{\sqrt{\mathrm{Im}\,\Phi}} \cdot \underbrace{\begin{pmatrix}\boxminus & \boxminus\\ \boxminus & \boxminus\\ \boxminus & \boxminus\end{pmatrix}}_{\gamma^T} \cdot \underbrace{\begin{pmatrix}\boxdot & \boxdot\\ \boxdot & \boxdot\end{pmatrix}}_{G} \cdot \underbrace{\begin{pmatrix}\boxminus & \boxminus & \boxminus\\ \boxminus & \boxminus & \boxminus\end{pmatrix}}_{\gamma} \cdot \underbrace{\begin{pmatrix}\boxplus & 0 & 0\\ 0 & \boxplus & 0\\ 0 & 0 & \boxplus\end{pmatrix}}_{\sqrt{\mathrm{Im}\,\Phi}},$$

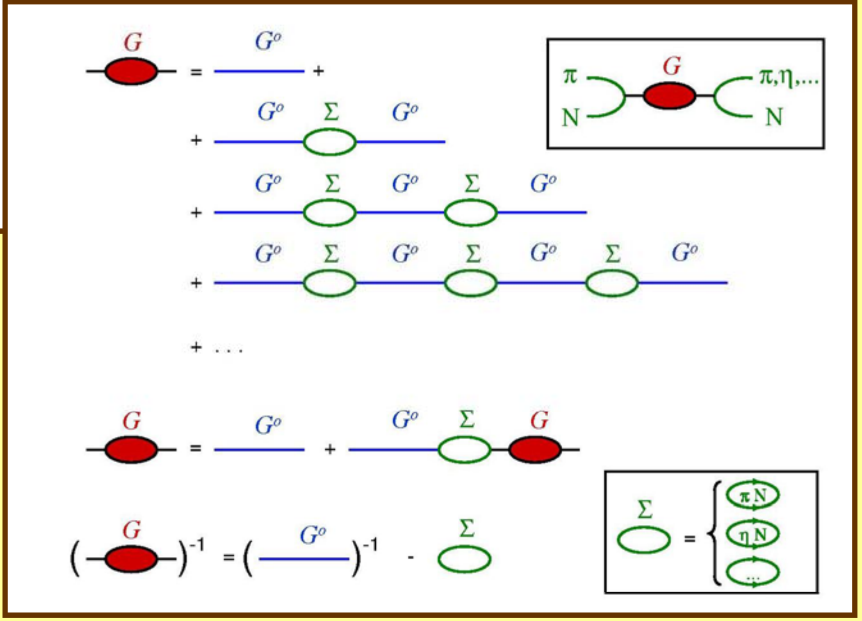
$\boxtimes$ experimentally obtained

$\boxplus$ theory-produced functions

$\boxminus$ parameterization by elementary functions

$\boxdot$ functions from other sources

$$\underbrace{\begin{pmatrix}\boxdot & \boxdot\\ \boxdot & \boxdot\end{pmatrix}}_{G^{-1}} = \underbrace{\begin{pmatrix}\boxminus & 0\\ 0 & \boxminus\end{pmatrix}}_{(G^{\circ})^{-1}} - \underbrace{\overbrace{\begin{pmatrix}\boxminus & \boxminus & \boxminus\\ \boxminus & \boxminus & \boxminus\end{pmatrix}}^{\gamma} \cdot \overbrace{\begin{pmatrix}\boxplus & 0 & 0\\ 0 & \boxplus & 0\\ 0 & 0 & \boxplus\end{pmatrix}}^{\Phi} \cdot \overbrace{\begin{pmatrix}\boxminus & \boxminus\\ \boxminus & \boxminus\\ \boxminus & \boxminus\end{pmatrix}}^{\gamma^T}}_{\Sigma},$$

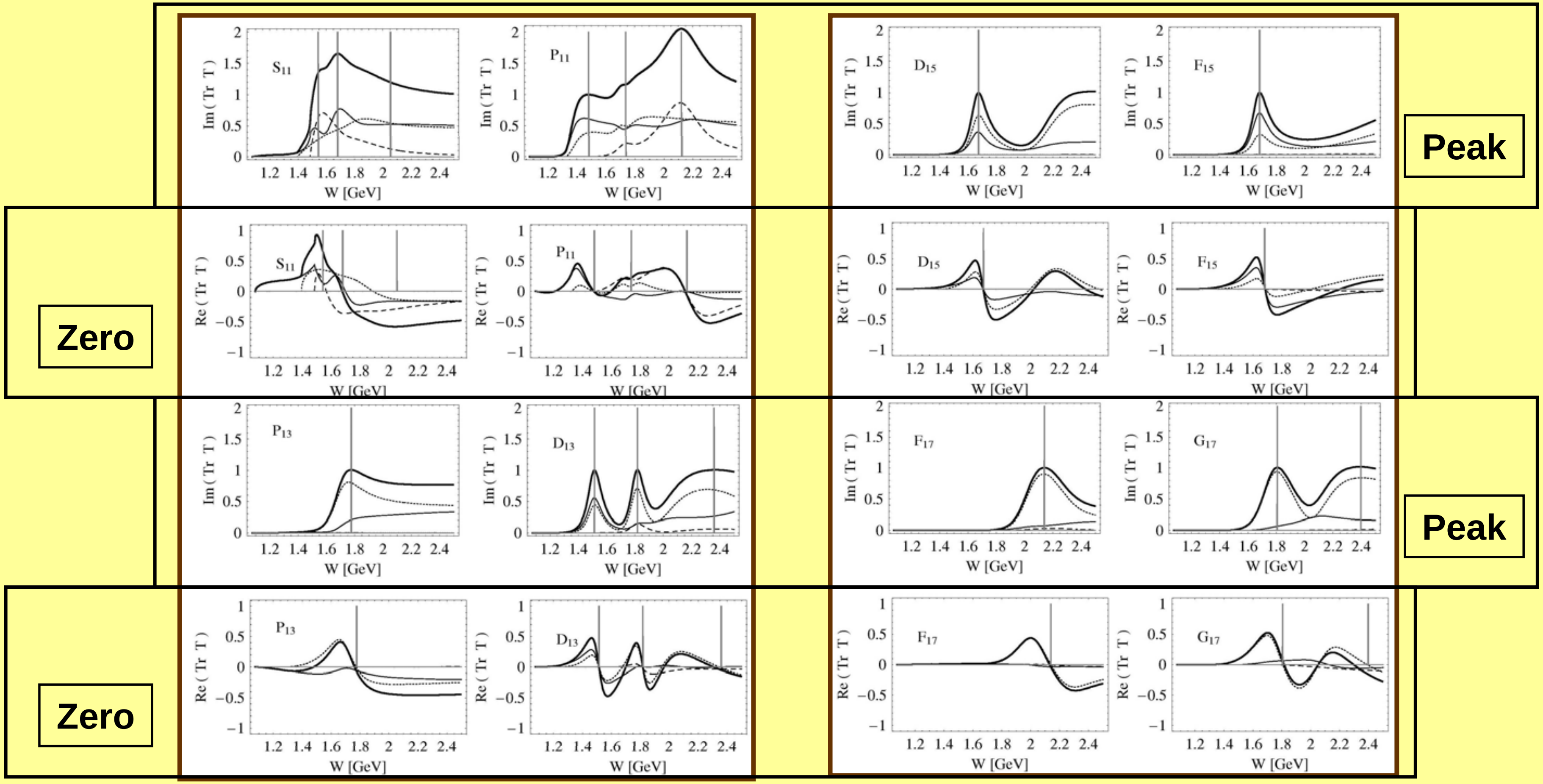


K-matrix pole parameters – **values**

It seems like there is **some relation** between BW and K-matrix pole parameters?

L_{2I2J} $\left[\begin{matrix} x_{\pi N}/x_{\eta N}/x_{\pi^2 N} \\ M/\Gamma \end{matrix}\right]$ PDG [1]	M^R (MeV)	Γ^R (MeV)	$x_{\pi N}$ (%)	$x_{\eta N}$ (%)	$x_{\pi^2 N}$ (%)
S_{11} $\left[\begin{matrix} 35-55/30-55/1-10 \\ 1535\pm^{20}_{15}/150\pm 50 \end{matrix}\right]$	**1543** 1553	**165** 182	**39** 46	**54** 50	**7** 4
S_{11} $\left[\begin{matrix} 55-90/3-10/10-20 \\ 1650\pm^{30}_{10}/150\pm^{40}_{5} \end{matrix}\right]$	**1680** 1652	**233** 202	**64** 79	**16** 13	**20** 8
S_{11} $\left[\begin{matrix} \text{NPV} \\ \approx 2090/\text{NPV} \end{matrix}\right]$	**2054** 1812	**1926** 405	**47** 32	**3** 22	**50** 46
P_{11} $\left[\begin{matrix} 60-70/0/30-40 \\ 1440\pm^{30}_{10}/350\pm 100 \end{matrix}\right]$	**1482** 1439	**541** 437	**61** 62	**0** 0	**39** 38
P_{11} $\left[\begin{matrix} 10-20/6/40-90 \\ 1710\pm 30/100\pm^{150}_{50} \end{matrix}\right]$	**1738** 1740	**170** 140	**44** 28	**12** 12	**44** 60
P_{11} $\left[\begin{matrix} \text{NPV} \\ \approx 2100/\text{NPV} \end{matrix}\right]$	**2123** 2157	**379** 355	**3** 16	**83** 83	**14** 1
P_{13} $\left[\begin{matrix} 10-20/0/>70 \\ 1720\pm^{30}_{70}/100\pm 50 \end{matrix}\right]$	**1776** 1720	**409** 244	**20** 18	**0** 0	**80** 82
D_{13} $\left[\begin{matrix} 50-60/0/40-50 \\ 1520\pm^{10}_{5}/120\pm^{15}_{10} \end{matrix}\right]$	**1515** 1522	**121** 132	**56** 55	**0** 0	**44** 45
D_{13} $\left[\begin{matrix} 50-60/0/40-50 \\ 1700\pm 50/100\pm 50 \end{matrix}\right]$	**1818** 1817	**126** 134	**15** 9	**15** 14	**70** 77
D_{13} $\left[\begin{matrix} \text{NPV} \\ \approx 2080/\text{NPV} \end{matrix}\right]$	**2359** 2048	**1216** 529	**26** 17	**6** 8	**68** 75
D_{15} $\left[\begin{matrix} 40-50/0/50-60 \\ 1675\pm^{10}_{5}/150\pm^{30}_{10} \end{matrix}\right]$	**1674** 1679	**144** 152	**36** 35	**0** 0	**64** 65
F_{15} $\left[\begin{matrix} 60-70/0/30-40 \\ 1680\pm^{10}_{5}/130\pm 10 \end{matrix}\right]$	**1682** 1680	**144** 142	**67** 67	**1** 0	**32** 33
F_{17} $\left[\begin{matrix} \text{NPV} \\ \approx 1990/\text{NPV} \end{matrix}\right]$	**2139** 2262	**412** 2036	**7** 3	**3** 2	**90** 95
G_{17} $\left[\begin{matrix} 10-20/\text{NPV} \\ 2190\pm^{10}_{90}/450\pm 100 \end{matrix}\right]$	**1806** –	**286** –	**6** –	**0** –	**94** –
G_{17} $\left[\begin{matrix} -/-/- \\ -/- \end{matrix}\right]$	**2397** 2125	**1217** 381	**16** 18	**0** 0	**84** 82

Correlation – K-poles and T-trace

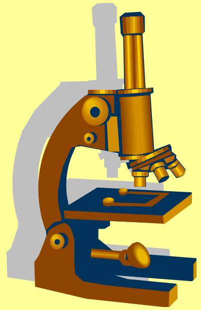


T-matrix poles: comparison of regularization and analytic continuation.

Review of Particle Physics [1]				AC		RM			
Particle	$L_{2I\,2J}$	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)	$\mathrm{Re}\,\mu$ (MeV)	$-2\,\mathrm{Im}\,\mu$ (MeV)	$\|r\|$ (MeV)	$-\theta$ (°)
$N(1535)$	S_{11}	1505(10)	170(80)	1517	190	1522	146	19	146
$N(1650)$	S_{11}	1660(20)	160(10)	1642	203	1647	203	84	58
$N(2090)$	S_{11}	N/E	N/E	1785	420	-	-	-	-
$N(1440)$	P_{11}	1365(20)	210(50)	1359	162	1354	162	47	95
$N(1710)$	P_{11}	1720(50)	230(150)	1728	138	1729	150	52	156
$N(????)$	P_{11}	N/E	N/E	1708	174	-	-	-	-
$N(2100)$	P_{11}	N/E	N/E	2113	345	2120	347	31	59
$N(1720)$	P_{13}	1700(50)	250(140)	1686	235	1691	235	19	112
$N(1520)$	D_{13}	1510(5)	115(5)	1505	123	1506	124	36	14
$N(1700)$	D_{13}	1680(50)	100(50)	1805	130	1806	132	7	36
$N(2080)$	D_{13}	N/E	N/E	1942	476	-	-	-	-
$N(1675)$	D_{15}	1660(5)	140(15)	1657	134	1658	138	25	20
$N(2200)$	D_{15}	N/E	N/E	2133	437	2145	439	22	71
$N(1680)$	F_{15}	1670(5)	120(15)	1664	134	1666	136	45	26
$N(1990)$	F_{17}	N/E	N/E	1990	303	2016	318	8	25
$N(????)$	G_{17}	N/E	N/E	1740	270	1749	280	6	86
$N(2190)$	G_{17}	2050(100)	450(100)	2060	393	2068	389	34	30

Interesting is that **strong discrepancies** occur at the **same places** as in the case of the **K-matrix poles**.

Some results of this research



Physics:

- **two extraction methods are developed** (regularization method i trace method),
- **correlation** between pole positions of K matrix and trace of matrix T is **quite strong** (but also **unexpected**),
- **stronger departures** from this correlation indicated (practically always) to known issues of the **original analysis**,
- we expect that this could be improved by addition of a few additional important inelastic channels,
- projector matrices χ most likely **do not have** resonant poles.

Mathematics:

- **a method is developed** that determines simple poles or zeros of complex functions (verified on various independent cases),
- **projector matrices** χ are introduced as a basis for expansion of the normal matrices.

Influence to standard approaches:

- **origin** and **misinterpretations** are given of the **speed plot** method (similarly for the **time delay**),
- critics is drawn on the **model-dependent methods** (like various **BW parameterizations** and **hybrid** approaches)
- most methods are developed for **narrow resonances** – in baryon physics they are simply **not narrow enough**.