The η-Nucleon Scattering Length and Effective Range

Contributors

•Began with J. A. Niskanen and S. Wycech:

Are there η-Helium bound states?

Phys.Rev.C **52** (1995) 544

• Continued with S. Wycech:

The η-Nucleon Scattering Length and Effective Range

Phys. Rev. C **55** (1997)R2167

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η-Nucleon Scattering Length and Effective Range uncertainties.

Phys.Rev. C**71** (2005) 014001

Other papers on this topic

η-Deuterium Scattering

(with J.A.Niskanen and S.Wycech)

Phys. Rev. C **54** (1996) 1970

A coupled K-matrix description of the reactions $\pi N \to \pi N$, $\pi N \to \eta N$, $\gamma N \to \pi N$ and $\gamma N \to \eta N$

(with S.Wycech)

Phys. Rev. C **60** (1999) 035208

Uncertainties in the η-Nucleon Scattering Length and Effective Range

(with S.Wycech)

nucl-th/0009053 – unpublished – more later

Why be interested in the η–N interaction?

- There are no η-beams.
- Useful for chiral and other effective theories.
- Useful for $\eta - \eta'$ mixing and structure of η'.
- Needed for studying/predicting η–Nucleus quasi–bound states.

Private communication from S. Wycech ETA07 Peniscola May 2007

1) H. Machner sees a possible η - Al state of $\sim$12 MeV width, loosely bound.

2) Krushe sees (unpublished) anomalies in final state η -nucleus interactions in photo-η production.

History in 2005

Table 1: A selection of ηN- scattering lengths and effective ranges appearing in the literature.

Reaction or Method	Scattering Length(fm)	Effective range (fm)
[1]	0.25+i0.16	
[2]	0.27+i0.22	
$pn \to d\eta$ [3]	$\leq$0.3	
[4]	0.46(9)+i0.18(3)	
[5]	0.487 +i0.171	−6.060−i0.177
[6]	0.51+i0.21	
[7]	0.55(20)+i0.30	
[5]	0.577 +i0.216	−2.807 −i0.057
[8]	0.621(40)+i0.306(34)	
[9]	0.68+i0.24	
[10]	0.717(30)+i0.263(25)	
Coupled K-mats[11]	0.75(4)+i0.27(3)	−1.50(13) −i0.24(4)
$\eta d \to \eta d$ [12]	$\geq$0.75	
Coupled K-mats[13]	0.87+i0.27	
[14]	0.91(5)+i0.29(4)	
[15]	0.980+i0.37	
[16]	0.991 +i0.347	−2.081 −i0.81
Coupled K-mats[13]	1.05+i0.27	

References

[1] C. Bennhold and H. Tanabe, Nucl. Phys. **A350**, 625 (1991)

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[16] G. Penner and U. Mosel, Phys. Rev. C **66**, 055212 (2002).

Reactions used to extract an η-nucleon scattering length

The parameters in the following model are adjusted to get an optimal fit to

- $\pi N \to \pi N$,
- $\pi N \to \eta N$,
- $\gamma N \to \pi N$ and
- $\gamma N \to \eta N$

data in an energy range of about 100MeV or so each side of the η threshold.

- Need η–N interaction at $\sim$ 1460 MeV cw actual threshold of 1487 MeV.

The 20–30 MeV is made up from $\sim$10MeV from nucleon separation and $\sim$10MeV due to recoil of η–N pairs.

Sketch of coupled K-matrix parametrization

Describe data by T-matrix ($T_{\pi,\eta} = T_{\pi N,\eta N}$ etc.)

$$T = \begin{pmatrix} T_{\pi,\pi} & T_{\pi,\eta} & T_{\pi,\gamma} \\ T_{\eta,\pi} & T_{\eta,\eta} & T_{\eta,\gamma} \\ T_{\gamma,\pi} & T_{\gamma,\eta} & T_{\gamma,\gamma} \end{pmatrix}, \quad (1)$$

Given $T_{\eta\eta}(q_\eta)$ define $a,\ r_0,\ s$ as:

$$T_{\eta\eta}^{-1} + iq_\eta = 1/a + \frac{r_0}{2} q_\eta^2 + s q_\eta^4 \quad (2)$$

To solve $T = K + iKqT$

express $T_{i,j}$'s in terms of $A_{i,j}$'s

$$T_{\pi,\pi} = \frac{A_{\pi,\pi}}{1 - iq_\pi A_{\pi,\pi}} \quad \text{and} \quad T_{\eta,\pi} = \frac{A_{\eta,\pi}}{1 - iq_\eta A_{\eta,\eta}} \quad \text{etc} \quad (3)$$

Now express $A_{i,j}$'s in terms of $K_{i,j}$'s

$$A_{\pi,\pi} = K_{\pi,\pi} + iK_{\pi,\eta}^2 q_\eta/(1 - iq_\eta K_{\eta,\eta}),$$

$$A_{\eta,\pi} = K_{\eta,\pi}/(1 - iq_\pi K_{\pi,\pi}) \quad \text{etc} \quad (4)$$

Sketch continued

$$K's \longrightarrow A's \longrightarrow T's. \tag{5}$$

Parametrize K's as

$$K_{\alpha\beta} = B_{\alpha,\beta} + \Sigma_i \frac{\sqrt{\gamma_\alpha(i)\gamma_\beta(i)}}{E_i - E} \tag{6}$$

where $\alpha, \beta = \pi N,\ \eta N,\ \gamma N$

Free parameters:

- 2 energies $E_0,\ E_1$ near S-wave $\pi - N$ resonances N(1535), N(1650),

- 6 couplings to the resonant states $\gamma_\pi(0,1), \gamma_\eta(0,1), \gamma_\gamma(0)$ and $\gamma_3(0)$.

- 4 background parameters $B_{\eta,\eta}, B_{\pi,\eta}, B_{\gamma\eta}, B_{\gamma\pi}$ – only $B_{\eta,\eta}$ sizeable.

Minuit gives errors on all parameters.
Used to estimate errors on $a,\ r_0,\ s$.

Two refinements

1) $\gamma_3(0)$ is the small effect of $\pi\pi N$.
The $\pi\pi N$ channel is removed explicitly by making, for example, the replacement

$$K_{\eta\eta} \to B_{\eta\eta} + \frac{\gamma_\eta(0)}{E_0 - E} + +i\frac{K_{\eta 3} q_3 K_{3\eta}}{1 - iq_3 K_{33}}, \quad (7)$$

where q_3 is a three body phase space element.
This treatment of the $\pi\pi N$ channel is not an approximation.
It is purely for convenience.

2) Momentum dependence in $B_{\pi\eta}$
Replacement

$$[B]^{-1}_{\pi,\eta} \longrightarrow [K^{pot}(E)]^{-1}_{\pi,\eta} = [B]^{-1}_{\pi,\eta} + R_{\pi,\eta} q^2_\eta, \quad (8)$$

where the parameter $R_{\pi,\eta}$ can be estimated from a_0–meson exchange to be ~ 0.2 fm.

Data fitted

- $\pi N \to \pi N$ Arndt et al.
  23 data points in the centre-of-mass energy range $1369.2 \leq E_{\rm cm} \leq 1705.0$ MeV

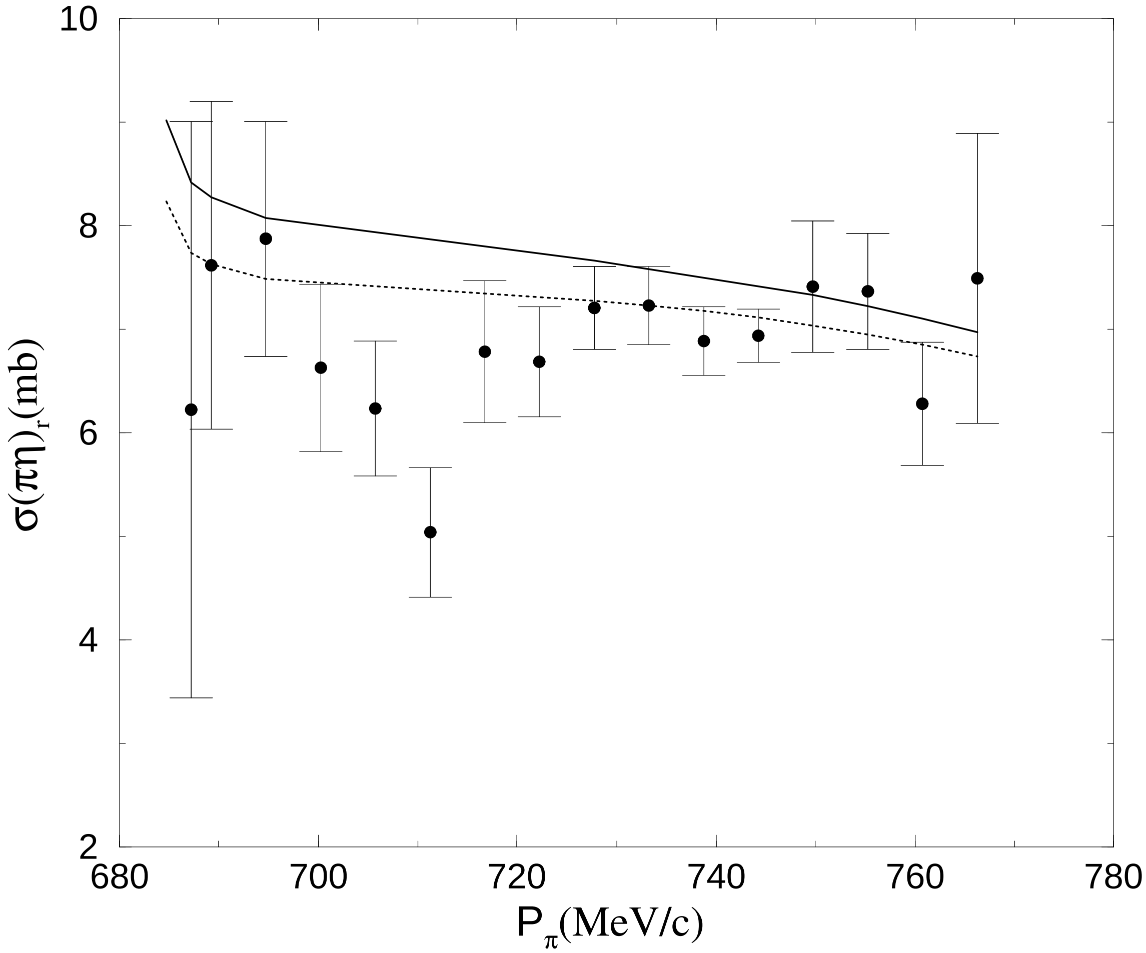
- $\gamma N \to \pi N$ Arndt et al.
  16 points with $1352.0 \leq E_{\rm cm} \leq 1546.3$ MeV

- $\gamma N \to \eta N$ Krusche and GRAAL
  38 points with $1487.0 \leq E_{\rm cm} \leq 1523.8$ MeV

- $\pi N \to \eta N$ Nefkens review and Brookhaven

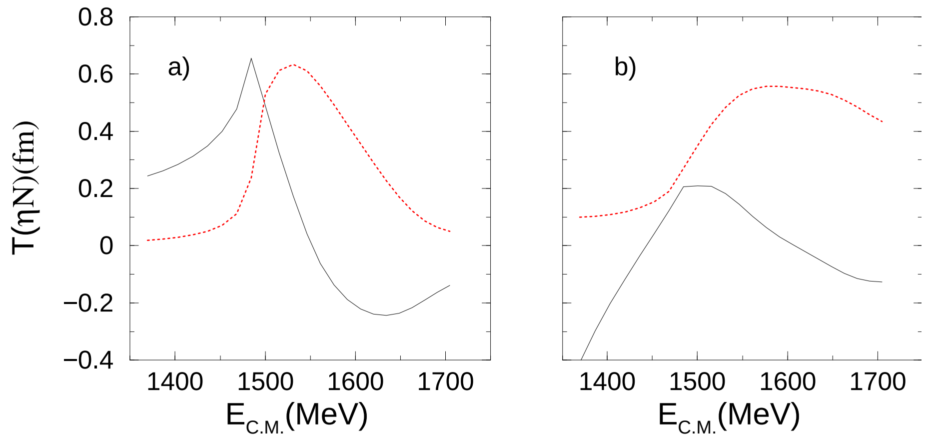
Preliminary $\pi N \to \eta N$ data

Figure 1: The reduced $\pi N \to \eta N$ cross section.
Preliminary from Tom Morrison (PhD thesis)
Solid line – Conventional solution
Dashed line – an Unconventional solution.



$\eta N \to \eta N$ AMPLITUDES

Figure 2: The $\eta N \to \eta N$ amplitudes
Solid line – Real part of $T_{\eta\eta}$:
Dotted, Imaginary part of $T_{\eta\eta}$:
a) Conventional b) Unconventional solutions
with essentially the same χ^2/dof



CONCLUSIONS

Either

K-matrix misses something in $\pi N \to \eta N$ channel

Or

This $\pi N \to \eta N$ data needs checking.

PRC referee agreed – we should wait.

No new scattering length.

Old value $a(fm) = 0.87 + i0.27$ favours the Conventional solution.

New $\pi N \to \eta N$ data

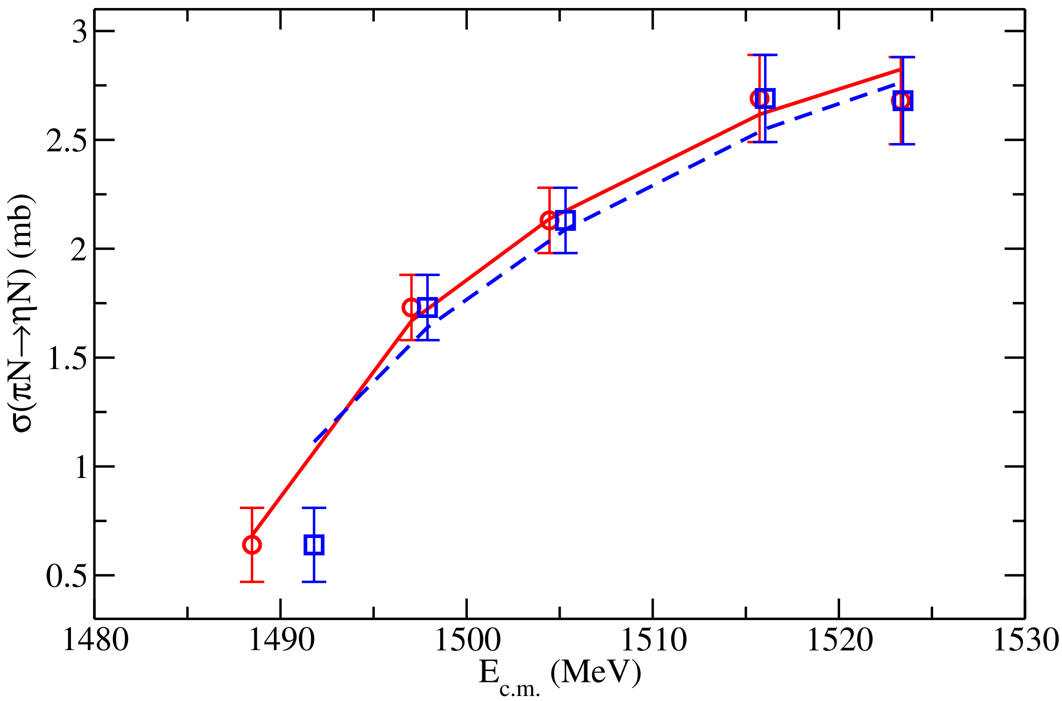
Table 2: The $\pi N \to \eta N$ data from Ref. below. The uncorrected π-beam momentum is denoted by [UC] and the corrected momentum by [C] – private communication V. Abaev.

P_π MeV/c [UC]	P_π MeV/c [C]	σ mb
692.5	687.1	0.64(17)
702.4	701.0	1.73(15)
714.5	713.1	2.13(15)
732.1	731.6	2.69(20)
744.3	744.1	2.68(20)

N. G. Kozlenko *et al.*, "Measurements of the Total and Differential Cross Sections of the Reaction $\pi^- p \to \eta n$ using the Polyethylene Target and the Crystal Ball Detector", Gatchina preprint 2542 (2003).

$\pi N \to \eta N$ Data 2

Figure 3: Fits to the $\pi N \to \eta N$ data with(solid line) and without(dashed line) the π-beam correction.
The squares are for the uncorrected π-beam momenta and the circles for the corrected momenta see Table 2.



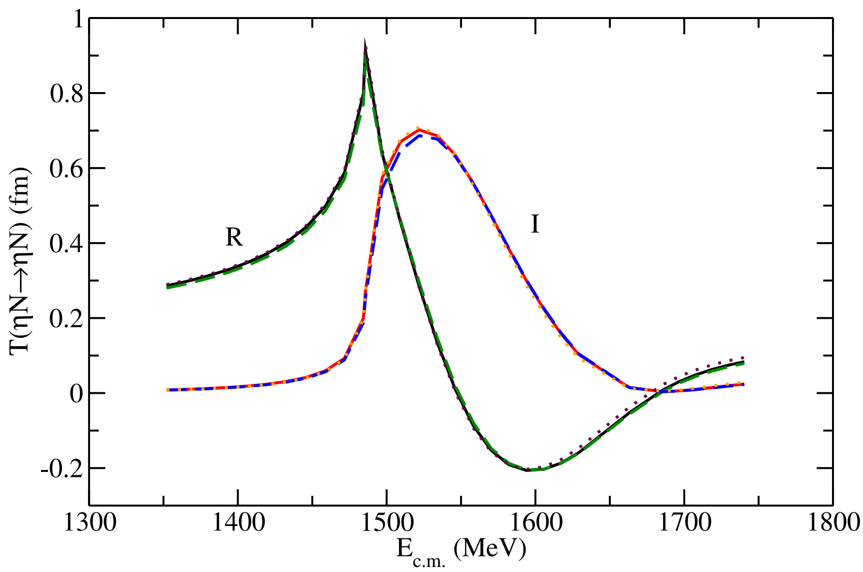
$\eta N \to \eta N$ AMPLITUDES

Figure 4: The real(R) and imaginary(I) parts of the $T(\eta\text{-}N)$ scattering amplitude.

The solid(dashed) lines show the fit with(without) the pion beam correction of the $\pi N \to \eta N$ data.

The dotted lines show the effect of not including this $\pi N \to \eta N$ data in the fit.

All three sets of curves are more or less indistinguishable.



Games with Data

Table 3: Effective range parameters a, r_0 in fm. Errors from parameter errors.

Re a	Im a	Re r_0	Im r_0
All	Data	Final	Result
0.91(6)	0.27(2)	–1.33(15)	–0.30(2)
All	Data	no beam	correction
0.88(5)	0.25(2)	–1.37(16)	–0.31(2)
No	$\pi N \to \eta N$		
0.93(21)	0.27(10)	–1.3(6)	–0.31(7)
No	$\gamma N \to \eta N$		
0.77(9)	0.25(5)	–1.8(4)	–0.3(1)
$K(\pi\eta)$	energy	dependent	
0.92(20)	0.27(9)	–1.3(6)	–0.30(6)

History Update

Table 4: A selection of ηN- scattering lengths and effective ranges appearing in the literature.

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$\eta d \to \eta d$ [12]	$\geq$0.75	
Coupled K-mats[13]	0.87+i0.27	
[14]	0.91(5)+i0.29(4)	
FINAL RESULT	0.91(6)+i0.27(2)	−1.33(15)-i0.30(2)
[15]	0.980+i0.37	
[16]	0.991 +i0.347	−2.081 −i0.81
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[14] M. Batinić, I. Šlaus, A. Švarc and B.M.K. Nefkens
Phys. Rev. C **51**, 2310 (1995), erratum Phys. Rev. C **57**, 1004 (1998).
Includes πN, ηN and effective $\pi\pi N$ channels – but no γN

Recent work

http://arxiv.org/abs/0704.1530

1) A study of the $pd \to pd\eta$ reaction

Authors: N. J. Upadhyay, K. P. Khemchandani, B. K. Jain, N. G. Kelkar

Large enhancement requires $\eta - d$ FSI corresponding to large $a_{\eta N}$
Around 1.07+i0.26fm
Better than (0.42, 0.34) fm or (0.75, 0.27) fm

2) Colin Wilkin(Peniscola) analyzed new η^3He data and obtained $a \sim \pm$ 10 $-i$1.5 fm.
If true that indicates very narrow state.
Probably not-true , depends on the experimental very controversial cuts.
That length disagrees with our η^3He length based on different data.
(Private communication S. Wycech)

A separable interaction

Useful in few-body problems to have a separable form for going off-shell:

$$T_{\eta\eta}(q, E, q') = v_\eta(q) t_{\eta\eta}(E) v_\eta(q'), \qquad (9)$$

where

$$v = 1/(1 + q^2\beta^2) \quad \text{with} \quad \beta = 0.31 \text{ fm} \qquad (10)$$

estimated roughly from $N(1535)$ formfactor and nearest singularities in t and u channels.

Conclusion

$a = 0.91(6) + i0.27(2)$,

$r_0 = -1.33(15)$-$i0.30(2)$,

$s = -0.15(1) - i0.04(1)$

a is in agreement with Alfred Švarc