Forward analysis of pion-nucleon scattering

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Outline

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Why Forward Analysis?

The Aim

- The ultimate aim: full PWA
- with fixed-t constraints.
- Needed: constraints for $t \in [0.0 \ldots -1.50]$ $(\text{GeV/c})^2$
- Expansion coefficients are continuous in t
- Optical theorem $\Rightarrow$ Forward direction special
- Forward direction is the starting point.

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The Expansion Method

Analyticity

- Invariant amplitudes are analytic functions
- $\Rightarrow$ Dispersion relations like

$$\mathrm{Re}\, C^+(\nu, t) = C_N^+(\nu, t) + \frac{2\nu^2}{\pi} \mathcal{P}\!\!\int_{\nu_T}^{\infty} \frac{d\nu'}{\nu'} \frac{\mathrm{Im}\, C^+(\nu', t)}{\nu'^2 - \nu^2} + C^+(0, t)$$

are satisfied, though unpractical.

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The Expansion Method

Analyticity

- Instead, Pietarinen's expansion is used:

$$C(\nu, t) = C_N(\nu, t) + H(Z, t) \sum_{k=1}^{N} c_k(t)[Z(\nu)]^k,$$

where

$$Z(\nu) = \frac{\alpha - \sqrt{\nu_T^2 - \nu^2}}{\alpha + \sqrt{\nu_T^2 - \nu^2}}.$$

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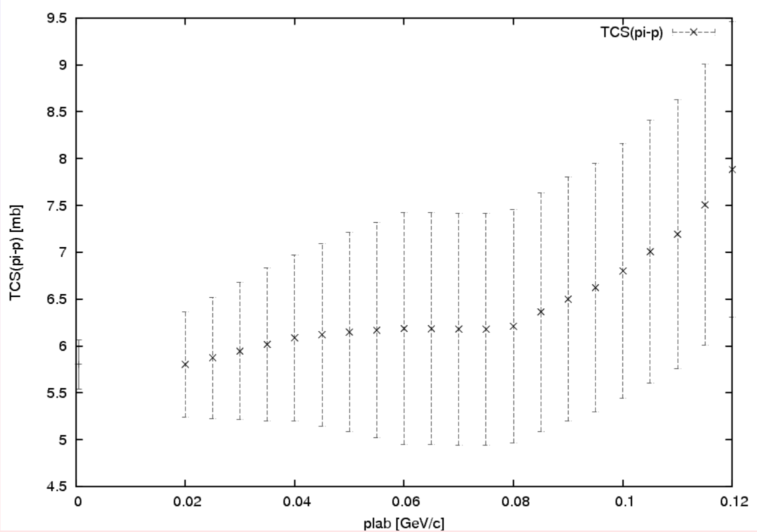
Forward Data

The input

- TCS,
- "partial total" cross sections,
- $\mathrm{Re}\, f / \mathrm{Im}\, f$,
- $\mathrm{Re}\, D^{+}$,
- scattering lengths.
- Low energy constraints.

Constraints



Minimization

Dedicated routine

- Fast.
- Sometimes unreliable.
- No covariance matrix.

Minuit

- Robust.
- 100 parameters $\Rightarrow$ not fast.
- 400 parameters and Hessian $\Rightarrow$ really slow.

Minimization

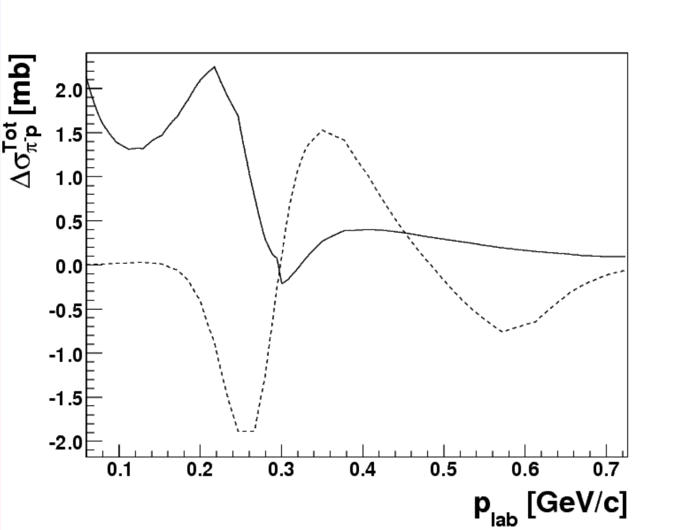
The Forward Case

- 200 coefficients,
- 136-142 data sets,
- $\Rightarrow$ 336-342 parameters.

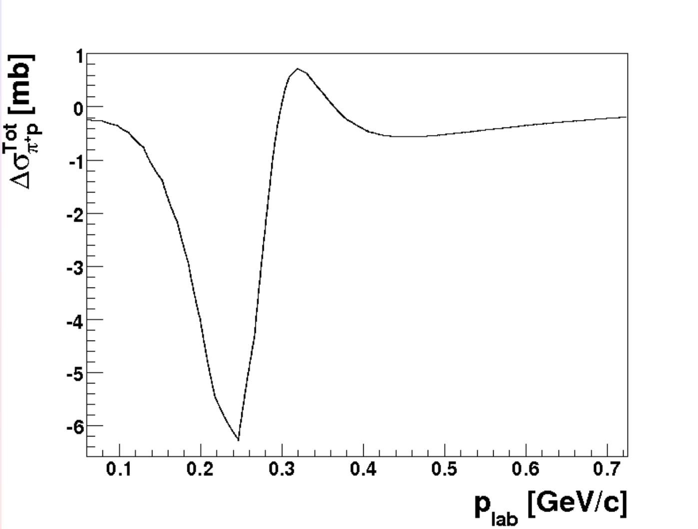
The iterations

- 1st iteration for Abaev $\Rightarrow$ Abaev's 1st DPSA.
- 2nd iteration for Abaev $\Rightarrow$ Abaev's 2nd DPSA.
- 3rd iteration for Abaev $\Rightarrow$ Abaev's 3rd DPSA.
- For J^- integral a new solution without Abaev's input.

EM Corrections for $\sigma^{\mathrm{Tot}}_{\pi^- p}$



EM Corrections for $\sigma^{\mathrm{Tot}}_{\pi^+ p}$



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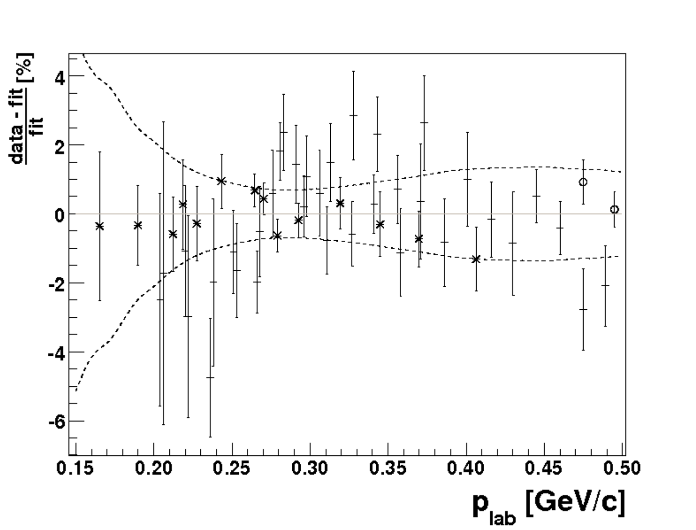
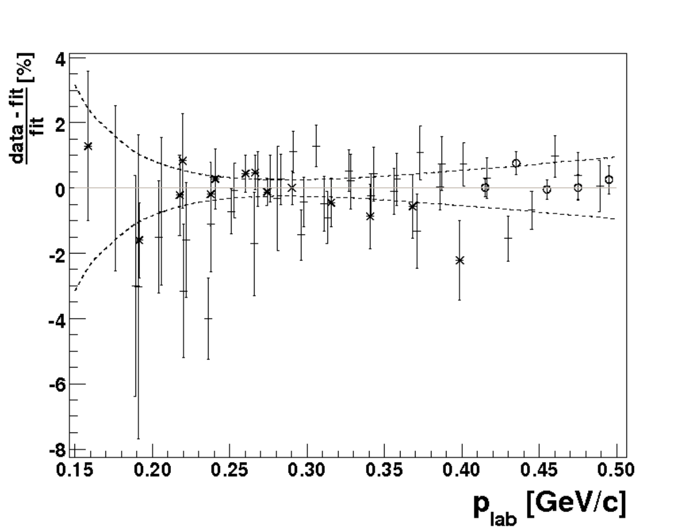
Difference Plot for $\sigma^{\mathrm{Tot}}_{\pi^- p}$

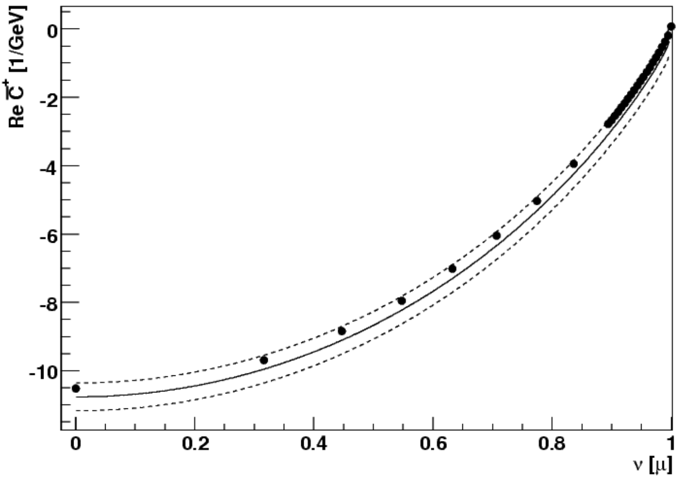


Difference Plot for $\sigma^{\mathrm{Tot}}_{\pi^+ p}$



(nyt rimpuilua läppärin kanssa)

Subthreshold Amplitude



Subthreshold parameters

$$c_{n0}^{+} = \frac{1}{n!}\sum_{k=1}^{N} c_k^{+} \frac{\partial^n}{\partial(\nu^2)^n}\left[Z^{k-1}H^{+}(Z)\right]\Big|_{\nu=t=0}$$

$$c_{n0}^{-} = \frac{g^2\delta_{n0}}{2m^2} + \frac{1}{n!}\sum_{k=1}^{N} c_k^{-} \frac{\partial^n}{\partial(\nu^2)^n}\left[Z^{k-1}H^{-}(Z)/\nu\right]\Big|_{\nu=t=0}$$

$$(\Delta c_{n0})^2 = \sum_{k,l}\frac{\partial c_{n0}}{\partial c_k}\frac{\partial c_{n0}}{\partial c_l} V_{kl}$$

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Subthreshold parameters c_{n0}^{+}

	Present analysis		Karlsruhe	SM99
c_{00}^{+}	$-1.20^{a} \pm 0.004$	± 0.03	-1.46 ± 0.10	-1.26 ± 0.02
c_{10}^{+}	$1.119 \pm 0.001 \pm 0.002$		1.12 ± 0.02	1.11 ± 0.02
c_{20}^{+}	0.2015 ± 0.0005	± 0.0008	0.200 ± 0.005	0.20 ± 0.01
c_{30}^{+}	$0.0568 \pm 0.0003 \pm 0.0001$		-	-

[a] The value resulting from a calculation with isospin invariance.

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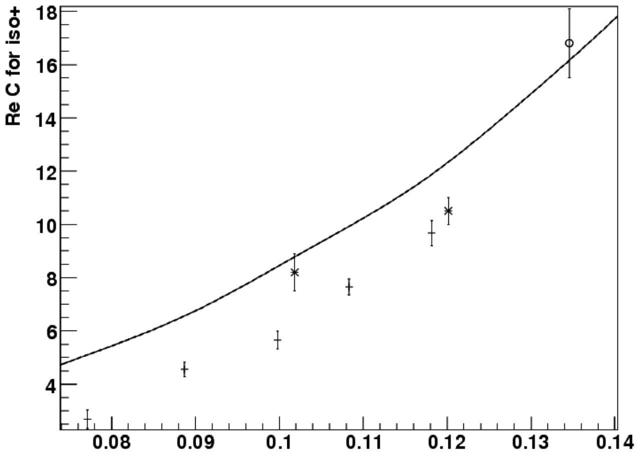
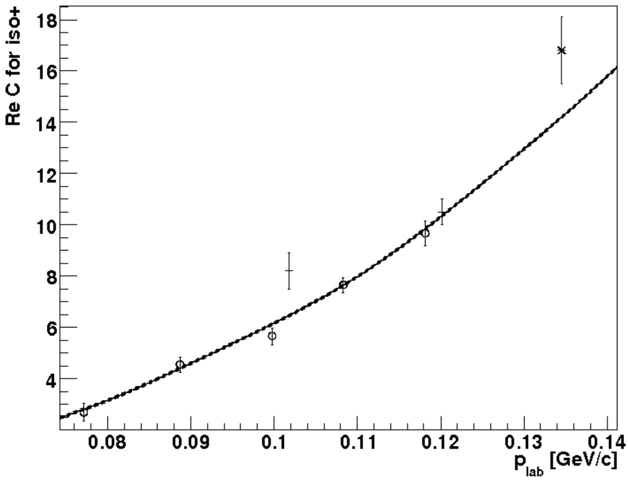
Denz data favours more negative c_{00}^{+}



Denz data favours more negative c_{00}^{+}



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Subthreshold parameters c_{n0}^{-}

	Present analysis	Karlsruhe
c_{00}^{-}	$1.41 \pm 0.002^{a} \pm 0.05^{b} \pm 0.01^{c}$	1.53 ± 0.02
c_{10}^{-}	$-0.167 \pm 0.001 \pm 0.001$	-0.167 ± 0.005
c_{20}^{-}	$-0.0388 \pm 0.0004 \pm 0.0005$	-0.039 ± 0.002
c_{30}^{-}	$-0.0092 \pm 0.0002 \pm 0.0001$	-

[a] The statistical error.
[b] The g^2 dependence.
[c] Due to conflicting data.

$$c_{n0}^{-} = \frac{g^2 \delta_{n0}}{2m^2} + \frac{1}{n!} \sum_{k=1}^{N} c_k^{-} \frac{\partial^n}{\partial(\nu^2)^n} \left[Z^{k-1} H^{-}(Z)/\nu \right] \Bigg|_{\nu=t=0}$$

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Partial Wave Analysis

The Actual Analysis

1. Pietarinen's expansions for all t-values
   - 150 t-values, 400 parameters for each
2. Partial Waves found by minimizing
   $\chi^2 = \chi^2_{\mathrm{data}} + \chi^2_{\mathrm{old}} + \chi^2_{\mathrm{ft}} + \chi^2_{\mathrm{unit}}$
   - Only the lowest waves
3. The higher PW's are from KA84