Effects of dispersion relation constraints on partial-wave amplitudes near the Roper resonance

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Outline

Introduction

- Abilene-Tuzla PWA based on KH methods
- Single-energy PWA with dispersion relation constraints
- Use recent data and perform a PWA with results suitable for input in resonance parameter extraction processes

Interactions of Interest

Scattering Interactions

$$\pi^+ p \longrightarrow \pi^+ p$$

$$\pi^- p \longrightarrow \pi^- p$$

$$\pi^- p \longrightarrow \pi^0 n$$

Fixed-Energy Partial Wave Analysis

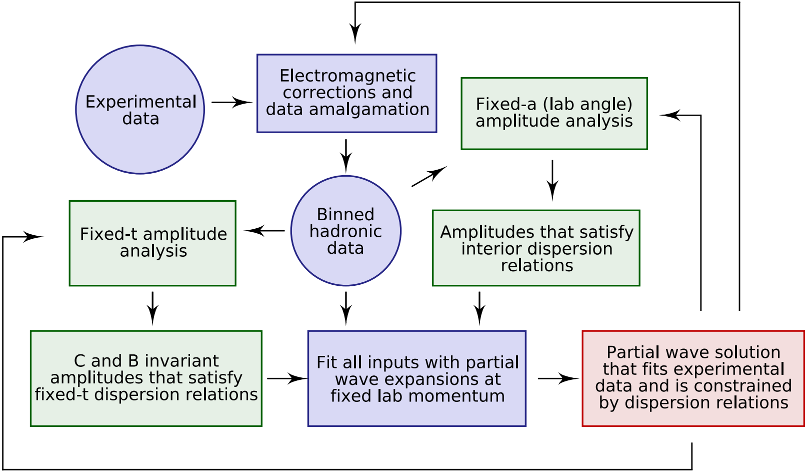
Fits to data are performed at fixed values of lab momentum

- Data are binned by lab momentum or comparable variable (s, W, T_π).
- Data points are shifted to the center of these bins using an existing partial wave analysis (using FA02 presently).
- Observables are calculated from terms of PW expansion and a least-squares function is minimized by varying the values of the partial-wave amplitudes.
- Due to differences in experimental data sets, solution from one bin may not be consistent with those of neighboring bins (e.g., normalizations).

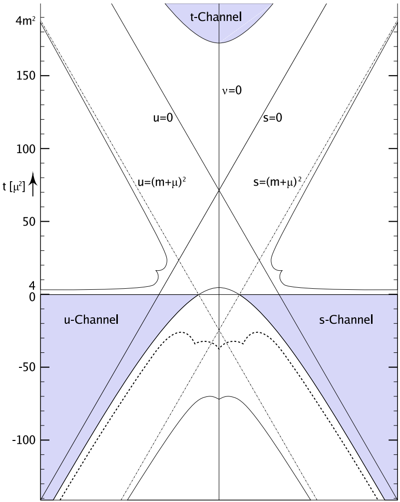
Why Perform this Partial Wave Analysis?

- Large amount of experimental data taken since KH80
- Computational capabilities improved dramatically since KH80
- Methods of Karlsruhe-Helsinki PWA can be applied to recent experimental data
- Extract resonance parameters using recent data as input

Fixed-Energy Start



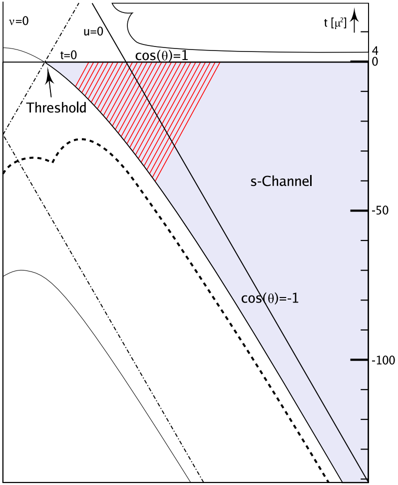
The Mandelstam Plane



Fixed-Energy Bins in the *s*-channel

The s-channel

- Fixed-energy bins are centered at fixed values of *s*
- The bins run perpendicular to the *s*-axis
- Fits here are smooth in $\cos\theta$



Fixed-Energy Analysis χ^2

- Data from available measurable quantities are fit simultaneously at a fixed value of lab momentum.
- KA84 is taken as the start solution, and higher order partial waves are fixed.
- One option: Data set normalization is optimized during the fit (not always reliable).

Function Minimized in Fixed-Energy PWA

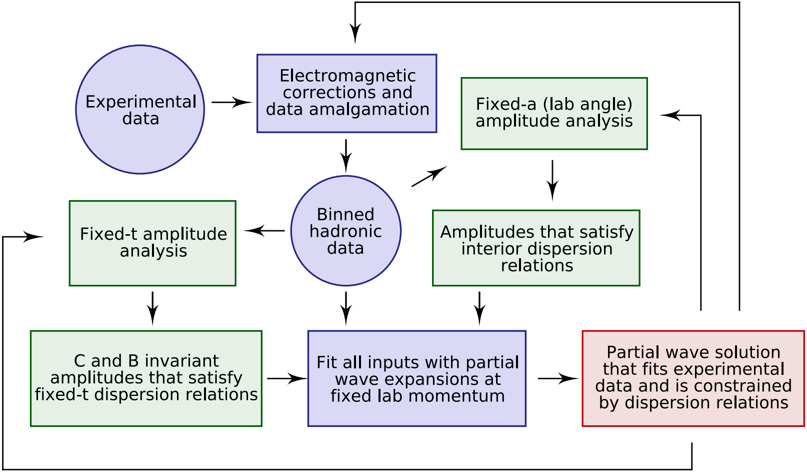
$$
\begin{aligned}
\chi^2 &= \chi^2_{\text{Data}} + \chi^2_{\text{Unitarity}} + \chi^2_{\text{Forward Amplitudes}} \\
&\quad + \chi^2_{\text{Normalization}} + \chi^2_{\text{Fixed}-t\text{, a Dispersion Relations}}
\end{aligned}
$$

Fixed Momentum Transfer Analysis

Purpose of this Fixed-t Amplitude Analysis

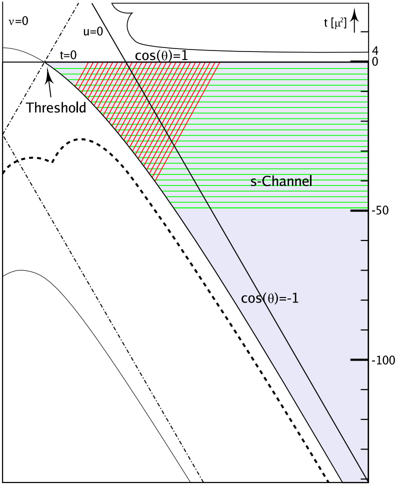
- Calculate analytic invariant amplitudes at fixed values of momentum transfer $\left(\sqrt{t}\right)$ that can be used to constrain a fixed-energy partial wave analysis
- An analysis down to $t = -1.0\,\text{GeV}^2$ could provide constraint for fixed-energy PWA in a region of$-1 < \cos\theta < 1$ and lab momentum up to approximately 800 MeV$/$c (currently, only implemented for $t \geq -0.45\,\text{GeV}^2$; ~500 MeV/c).
- The amplitudes should have analytic properties in the crossing variable ν $\left(\nu = \frac{s-u}{4m}\right)$
- The amplitudes should satisfy Mandelstam $s - u$ crossing symmetry
- The amplitudes should provide a good description of

Include Fixed-t Dispersion Relations



Fixed-t Bins in the s-channel

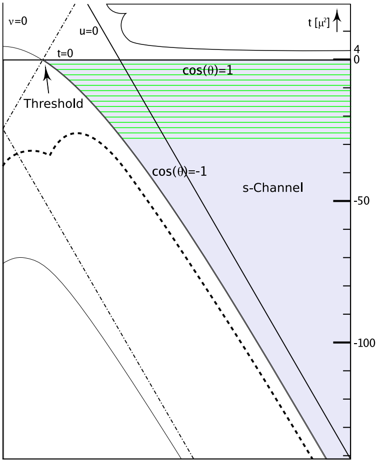
- Fixed-t bins are centered at fixed values of momentum transfer.
- The bins run parallel to the ν-axis.
- The fits here are analytic and smooth in ν.
- A fixed-energy analysis can be constrained by the fixed-t amplitude analysis at bin intersections.



Introduction

Analysis at Fixed t

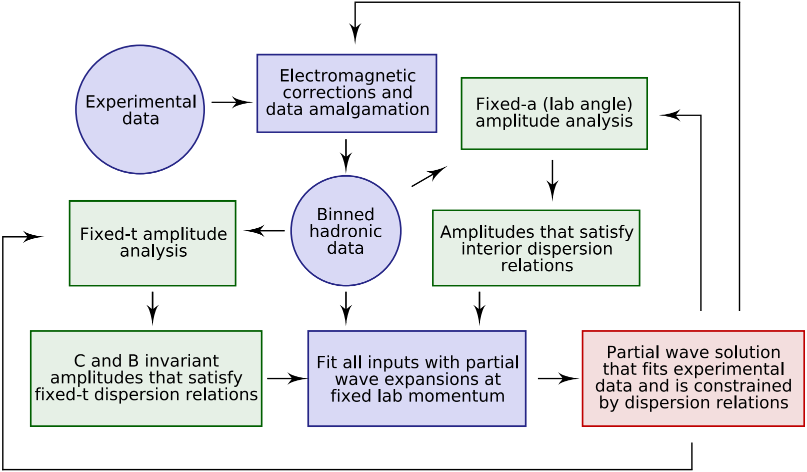
- Data are available in a large kinematic range (up to hundreds of GeV/c)
- Observables can be calculated from the invariant amplitude expansion and compared to data
- A least-squares minimization with penalty function to fit experimental data can be used to determine invariant amplitudes



Fixed-t Analysis

- Data from available measurable quantities are fit simultaneously at a fixed-t in terms of invariant amplitudes $C^{\pm}$ and $B^{\pm}$.
- The invariant amplitudes are expanded in terms of complex polynomial expansions with nucleon pole term subtractions and kinematic terms to account for asymptotic behavior.

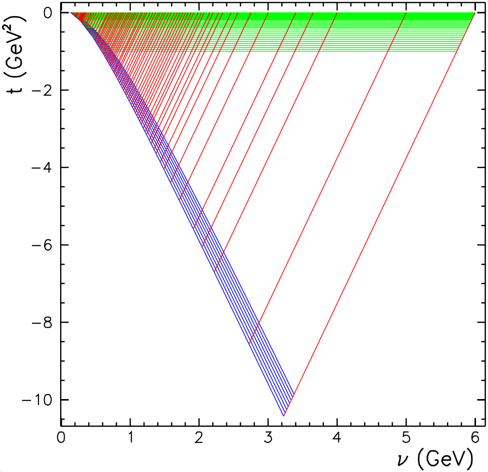
Simplified Process

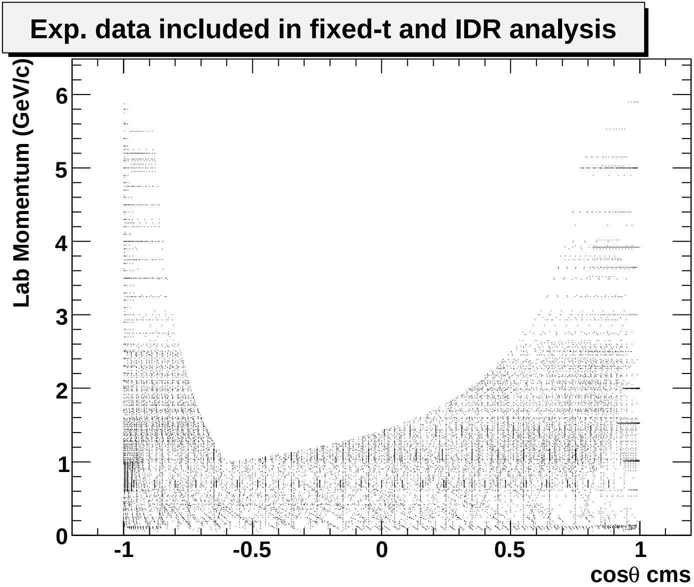


Including interior dispersion relation analysis

s-channel detail

- IDR bin locations added
- Fixed-*t* bin locations
- Single energy bin locations (fixed *s*)
- Mesh-like grid where analyses overlap
- Example shows coverage up to $p_{\mathrm{Lab}} = 6\,\mathrm{GeV/c}$





- Fixed-t to -1.0 GeV^2

Interior dispersion relation analysis (fixed-a)

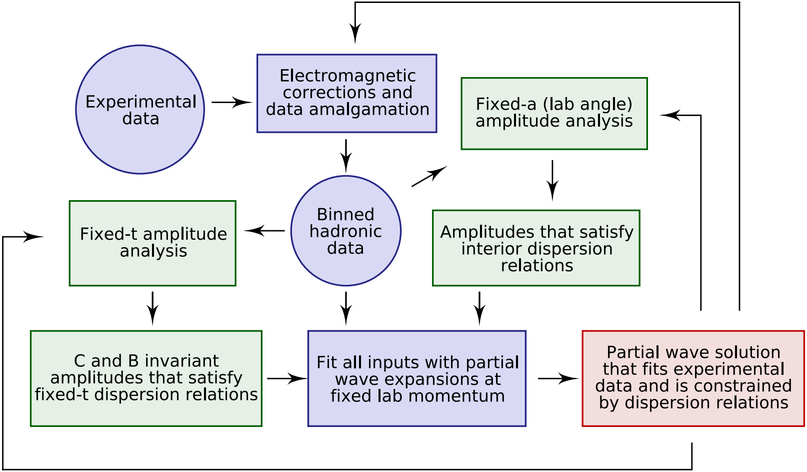
- IDR amplitude analysis was not used in KH80
- Amplitudes resulting from this analysis satisfy interior hyperbolic dispersion relations
- IDR analysis corresponds to fixed lab angle (θ_π)
- Covers lab angle from $180^\circ \leq \theta_\pi < 95^\circ$
- Covers a large kinematic region (up to high energy, $p_{\text{Lab}} \approx 25\,\text{GeV}/\text{c}$)
- Covers backward direction and complements region covered by fixed-t analysis

Difficulties when Fitting Experimental Data

Incomplete Experimental Data

- Data are not available everywhere in the kinematic regions of interest.
- There are gaps in angular distributions.
- There are gaps in energy.
- Only a relatively small amount of spin rotation parameter data are available.

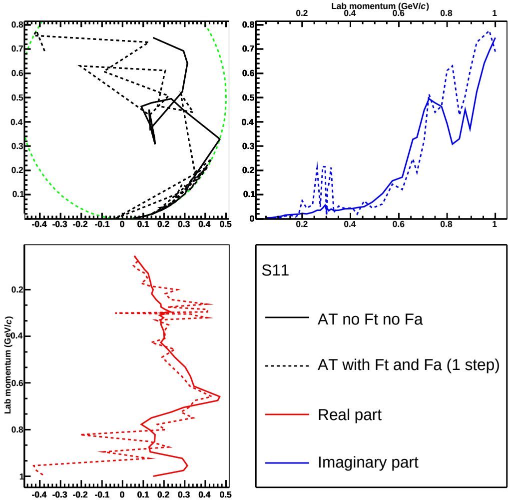
Simplified Process

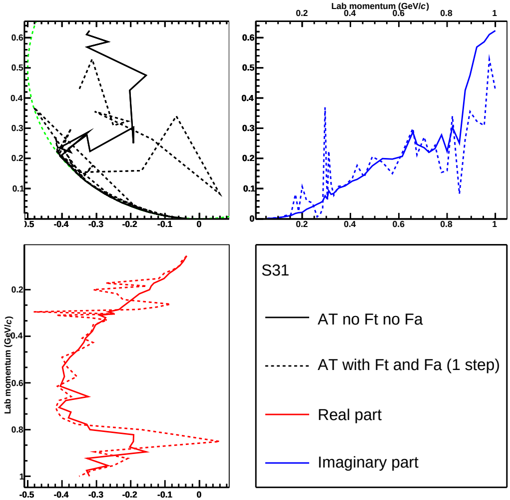


Difficulties when Fitting Experimental Data

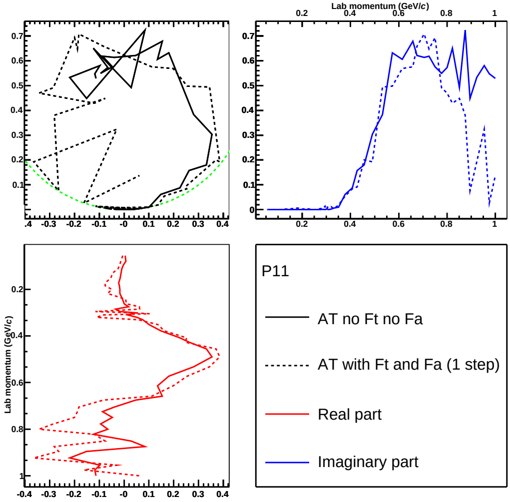
Disagreement Between Data Sets

- Normalization Differences
- Shape Differences

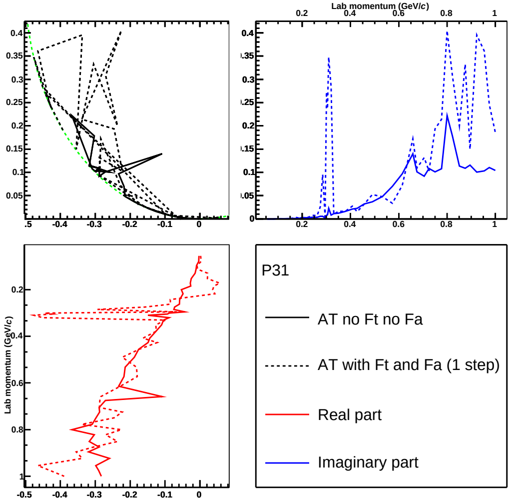

Lab momentum (GeV/c)
S11
AT no Ft no Fa
AT with Ft and Fa (1 step)
Real part
Imaginary part
Lab momentum (GeV/c)



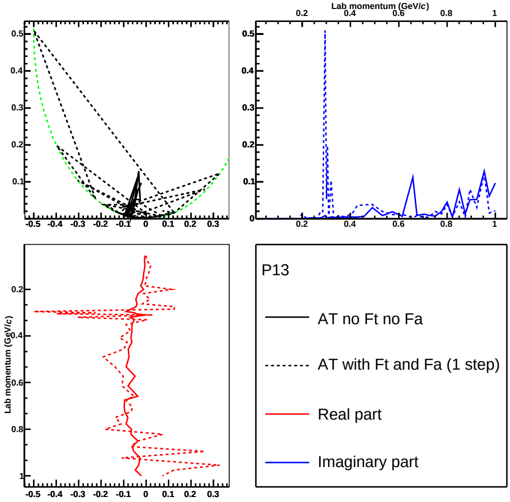
Lab momentum (GeV/c)
S31
AT no Ft no Fa
AT with Ft and Fa (1 step)
Real part
Imaginary part
Lab momentum (GeV/c)



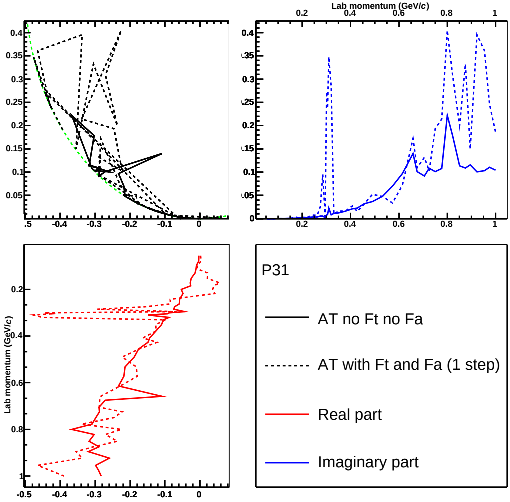
Lab momentum (GeV/c)
Lab momentum (GeV/c)
P11
AT no Ft no Fa
AT with Ft and Fa (1 step)
Real part
Imaginary part

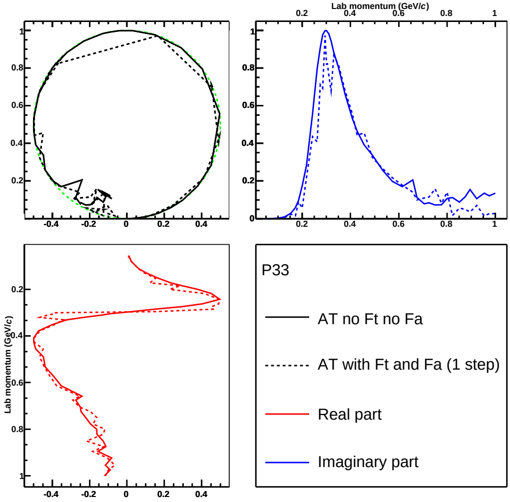


Lab momentum (GeV/c)
0.2
0.4
0.6
0.8
1
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.5
-0.4
-0.3
-0.2
-0.1
Lab momentum (GeV/c)
P31
AT no Ft no Fa
AT with Ft and Fa (1 step)
Real part
Imaginary part



Lab momentum (GeV/c)
P13
AT no Ft no Fa
AT with Ft and Fa (1 step)
Real part
Imaginary part
Lab momentum (GeV/c)


Lab momentum (GeV/c)
Lab momentum (GeV/c)
P31
AT no Ft no Fa
AT with Ft and Fa (1 step)
Real part
Imaginary part



Lab momentum (GeV/c)
Lab momentum (GeV/c)
P33
AT no Ft no Fa
AT with Ft and Fa (1 step)
Real part
Imaginary part

Summary

- A PWA suitable for resonance parameter extraction is highly desired.
- Constraints such as fixed-t and fixed-a dispersions should help eliminate ambiguities in the parameterization.
- An effective analysis should smooth statistical fluctuations and preserve features of physical significance (e.g., suitable for resonance parameter extraction).
- Currently, the implementation of the AT PWA is lacking and needs additional development.
- Consider adding additional constraints (e.g. fixed-θ_{CMS}). Modify coupling of different amplitude analyses with SE PWA. Explore dataset renormalization (handle consistently between different parts of the analysis).

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