Application of the model-independent method for T-matrix pole parameters extraction

...The end of Speed plot?

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A little bit of history

- Speed plot has been used and abused by many as a tool to extract resonance parameters.
- It is said to extract T-matrix pole parameters.

A little bit of history

- Hohler is the one who promoted the method, although he did not invent it.

  Herndon et al., Lawrence Radiation Laboratory Report 1960

- In years, lot of analyses depended on Speed Plot to extract their poles.

A little bit of history

- Everything was “peachy”, until Mike and Saša had an argument in Grenoble a couple of years back.
- The result of that argument was presented last year in Tuzla and for the sake of completeness, I’ll repeat it here.

The Argument

- So, what was the argument about?

- Speed plot produced different poles in different channels from Zagreb 1998 analysis.

The Argument

- Everything seems normal, except a tiny little thing:

- This analysis has poles on the same place in all channels by construction!

Speed Plot

$T = T_R + T_B$

$$T_R = \frac{r}{\mu - W}$$

$$\left|\frac{dT_R}{dW}\right| = \frac{|r|}{\left(\operatorname{Re}\mu - W\right)^2 + (\operatorname{Im}\mu)^2}$$

The method

- After a looong and agonizing procedure, a new method was born.

- The method had to have as few limitations as possible, and to be as general as possible.

The method

Now comes a small mathematical workout, in which we will see that simplicity and going by-the-book pays off...

Road to The method

$$f(z) = (\mu - z)\, t(z)$$

$$f(\mu) = \sum_{n=0}^{N} \frac{f^{(n)}(x)}{n!}(\mu - x)^n + R_N(x, \mu).$$

$$f^{(n)}(x) = (\mu - x)\, t^{(n)}(x) - n\ t^{(n-1)}(x),$$

$$|f(\mu)| = \frac{\left|t^{(N)}(x)\right|}{N!}\, |a + i\, b - x|^{(N+1)}\ .$$

Regularization method

$$\frac{(a-x)^2+b^2}{\sqrt[N+1]{|f(\mu)|^2}} = \sqrt[N+1]{\frac{(N!)^2}{|t^{(N)}(x)|^2}}.$$

- This is the final formula.

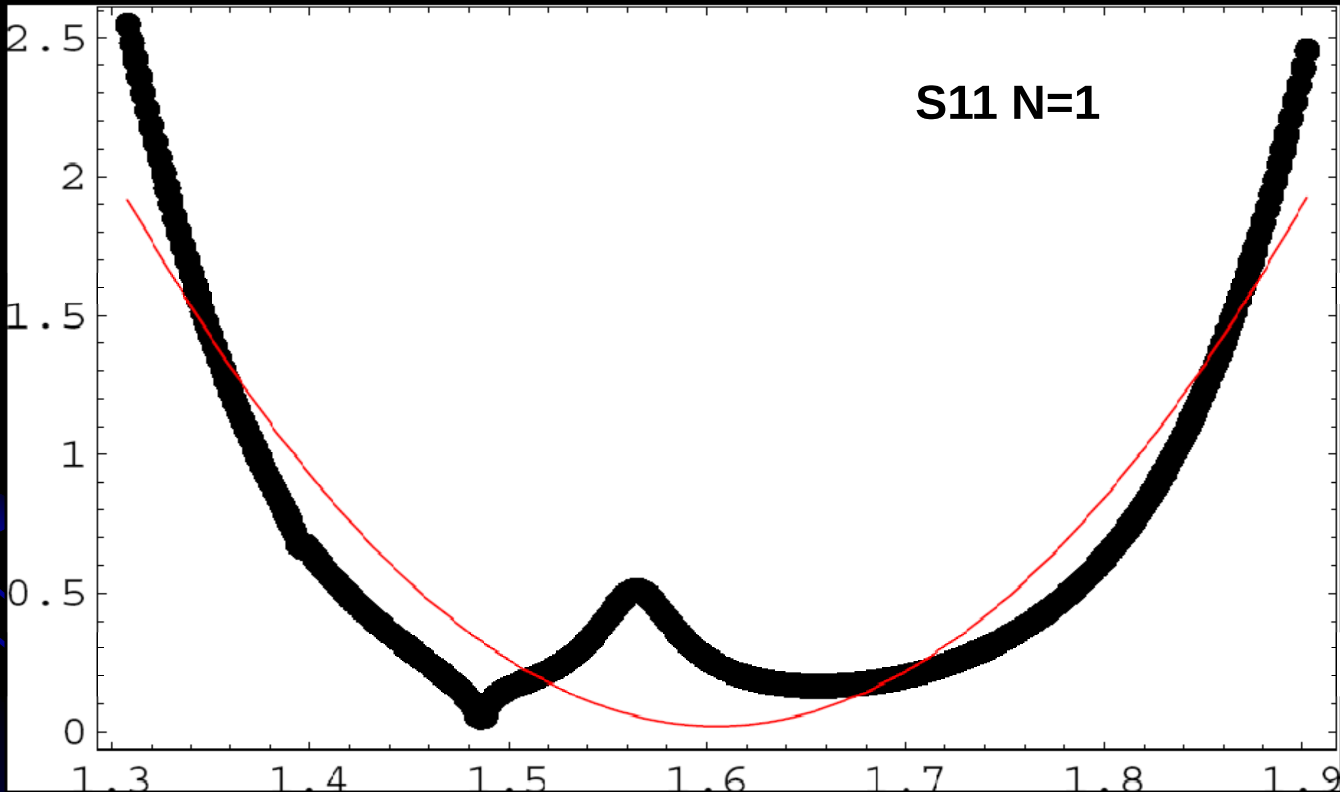
Pretty neat, isn't it?

Application

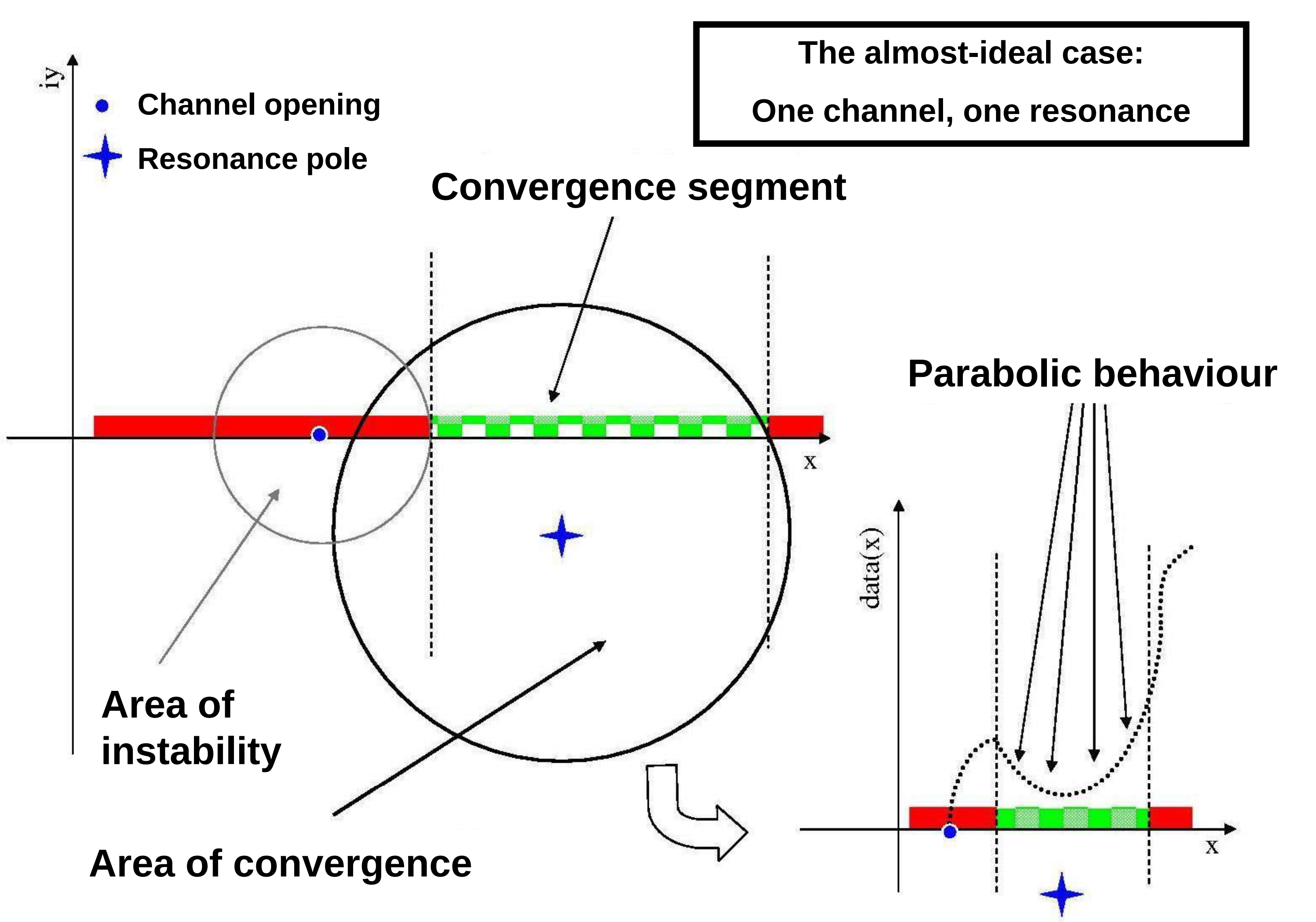
- Now, a method without application is like a cowboy without a horse.

- We applied the method on Zagreb 1998 analysis and got some interesting results

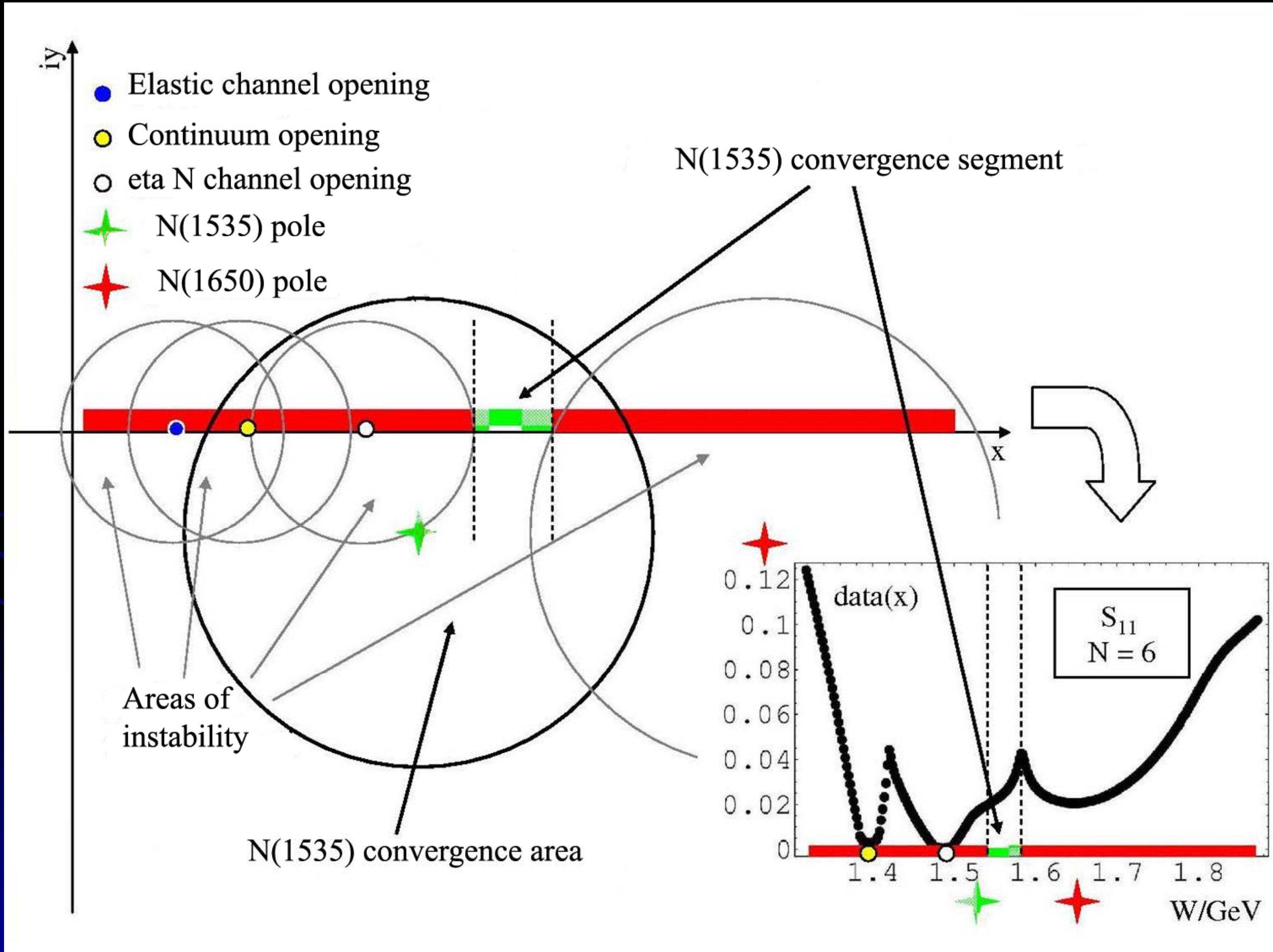
How does it work?



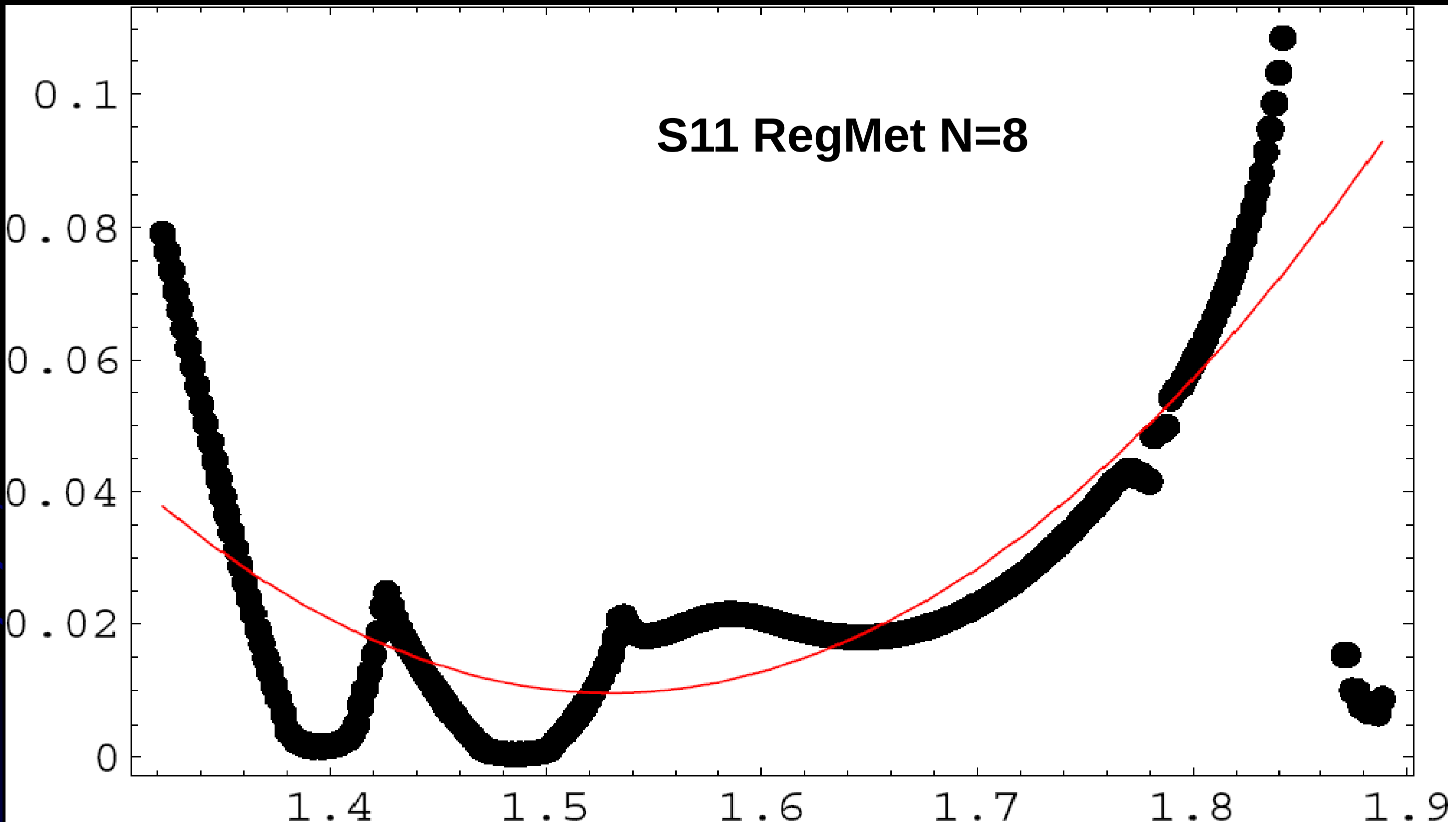
Application



Application II



How does it work?



Results

N(1535)S11	Speed Plot Re; -2Im;	RegMet Re; -2Im;
$\pi N \rightarrow \pi N$	1506;83;	(1522;146;)
$\pi N \rightarrow \eta N$	1531;388;	N.C.
Trace (T)	1525;82;	N.C.

Results

N(1650)S11	Speed Plot Re; -2Im;	RegMet Re; -2Im;
$\pi N \to \pi N$	1658;180	1646;205
$\pi N \to \eta N$	1601;208	1646;208
Trace (T)	1665;163	1648;202

Results

- To go on, it would be boring. Therefore, here is a self-explanatory table of N* resonances.

The Table

Resonance	Speed Plot ΔRe;Δ(-2Im)	RegMet ΔRe; Δ(-2Im)
N(1535)S11	25;300	N.C.
N(1650)S11	64;45	2;8
N(2090)S11	N.E. in ηN ch.	N.C.
N(1440)P11	S.T. In ηN ch.	S.T. In ηN ch.
N(1710)P11	100;54	5,16
N(2100)P11	15,38	1;1
N(1720)P13	89;79	1;1
N(1520)D13	1,16	1;1

The Table Cont.

Resonance	Speed Plot ΔRe;Δ(-2Im)	RegMet ΔRe; Δ(-2Im)
N(1700)D13	152;145	1;1
N(2080)D13	21;164	N.C.
N(1675)D15	39;41	1;1
N(2200)D15	19;32	1;3
N(1680)F15	1;1	1;1
N(1990)F17	39;96	2,6
N(????)G17	11;10	1;3
N(2190)G17	1;182	5;5

Let's go back to Speed Plot

$$\frac{(a-x)^2+b^2}{\sqrt[N+1]{|f(\mu)|^2}} = \sqrt[N+1]{\frac{(N!)^2}{|t^{(N)}(x)|^2}}.$$

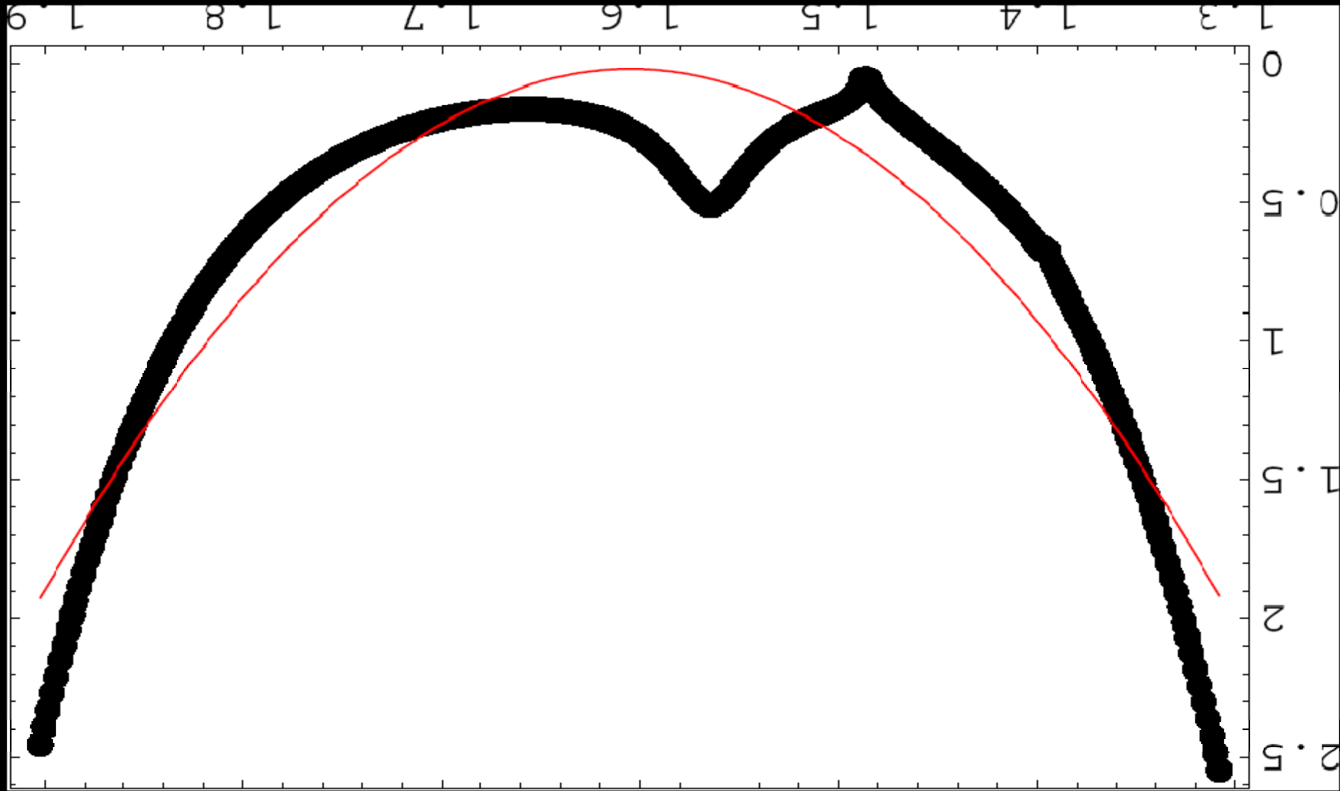
***N* = 1**

$$\frac{(a-x)^2+b^2}{|f(\mu)|} = \frac{1}{|dt(x)/dx|},$$

$$\left|\frac{dt(W)}{dW}\right| = \frac{|f(\mu)|}{(a-W)^2+b^2}.$$

$$\left|\frac{dT_R}{dW}\right| = \frac{|r|}{(\mathrm{Re}\,\mu - W)^2 + (\mathrm{Im}\,\mu)^2}$$

Example



That's why Hohler reported pole of N(1535) at the energy of ηN channel opening.

Why is it important?

Excited Baryon Program at JLab

A Contribution to the NSAC Long Range Plan

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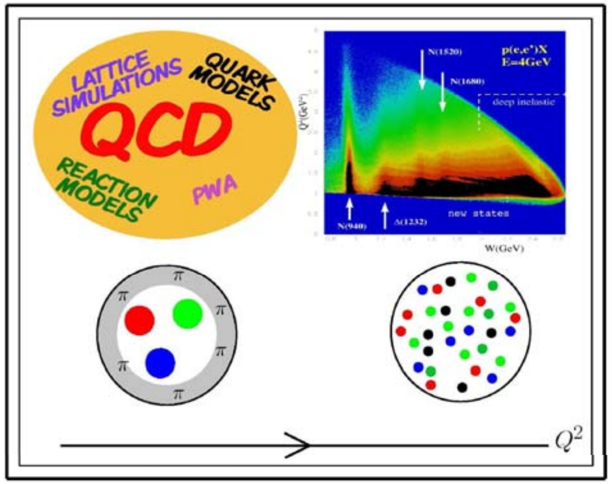

IV. EXCITED BARYON ANALYSIS CENTER

The Excited Baryon Analysis Center (EBAC) was established at JLab in January, 2006 to provide theoretical support to the excited baryon program. EBAC's program has two components. The first one is to identify new baryon states and extract the N^* parameters

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analyses must be examined. The validity of the often used speed-plot or time-delayed plot methods in extracting the N^* parameters from the determined partial-wave amplitudes should also be studied.

Conclusion

- Speed plot is just a first-order Regularization method
- Speed plot would give nice results if one would apply it to partial waves with poor analytical structure.
- Trace of T-matrix is a nice tool if analysis can produce one


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